The Complexity of Routing with Few Collisions

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21st International Symposium on Fundamentals of Computation Theory, September 11th, 2017
Bordeaux

Routing Autonomous Vehicles

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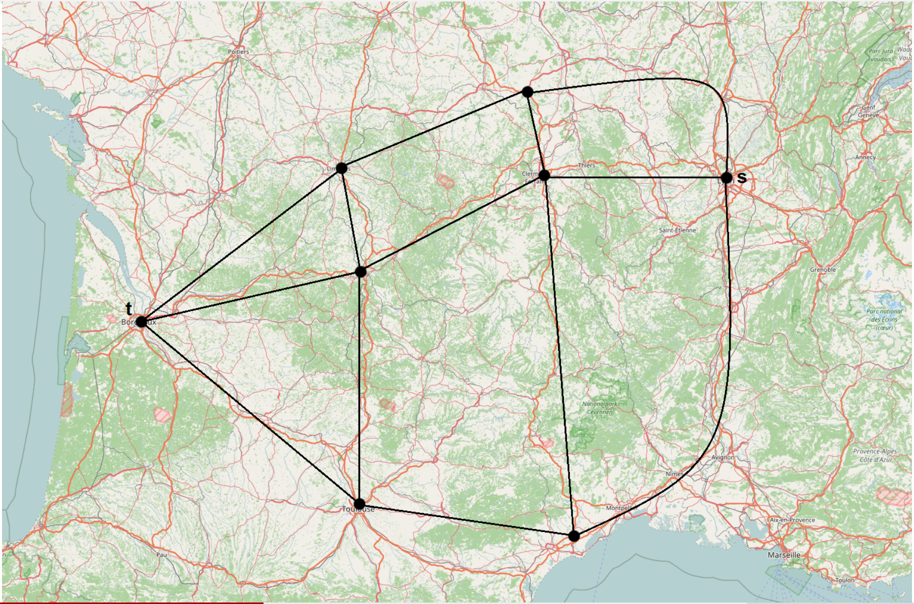

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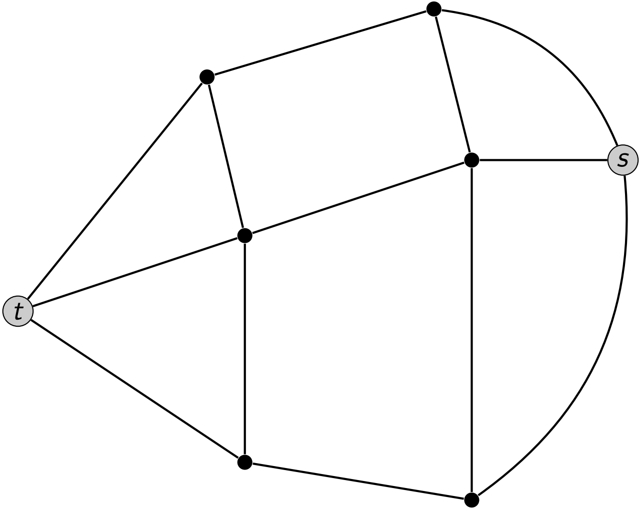

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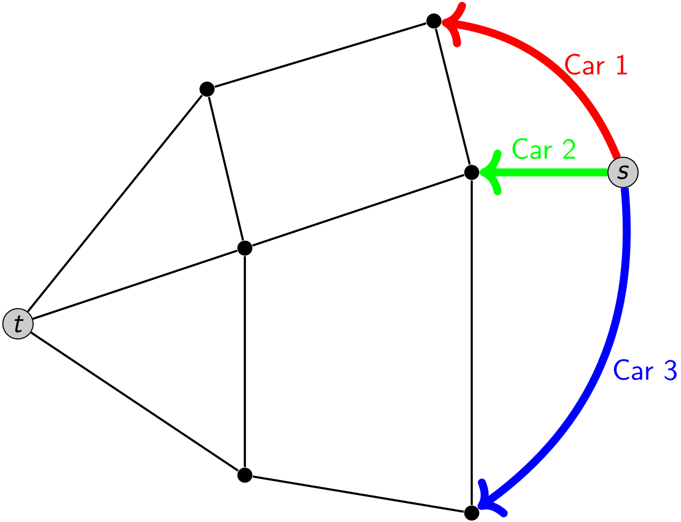

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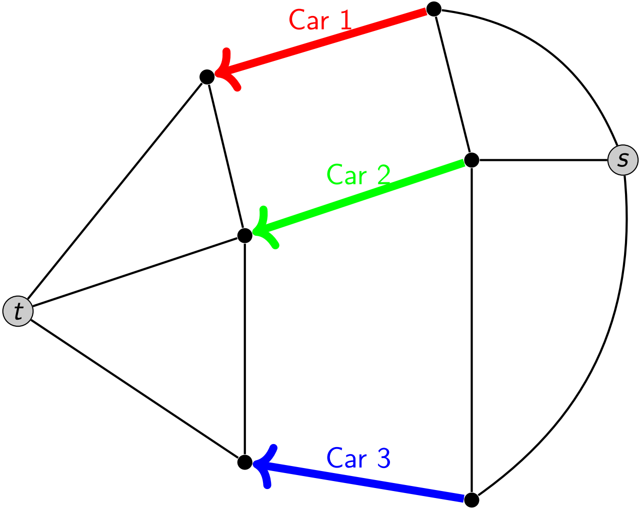

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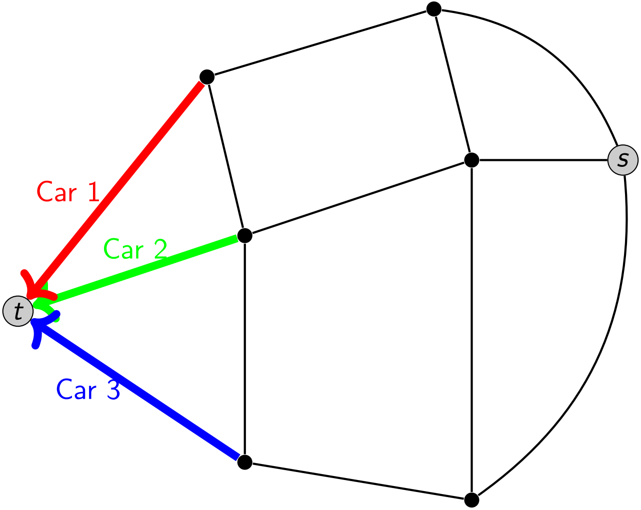

Problem Definition

Fast Routing with Collision Avoidance (FRCA)

Input: A graph $G = (V, E)$, source and sink $s, t \in V$, number of paths p, number of time-shared edges k, length ℓ.

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s-t-routes:

- **Walk**: Sequence of vertices: $P = (v_1, v_2, \ldots, v_n)$ s.t. $\forall i \in [1, n-1] : \{v_i, v_{i+1}\} \in E,\ v_1 = s,\ v_n = t$

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- **Path**: A walk with each vertex at most once

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An edge e is time-shared, if there are at least 2 s-t-routes in the solution of **FRCA** $P_1 = (v_1, \dots, v_a), P_2 = (u_1, \dots, u_a)$:
$\exists i : e = \{v_i, v_{i+1}\} = \{u_i, u_{i+1}\}$

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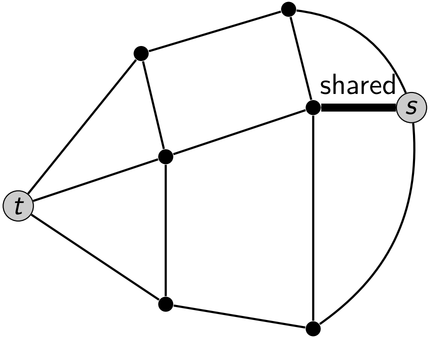

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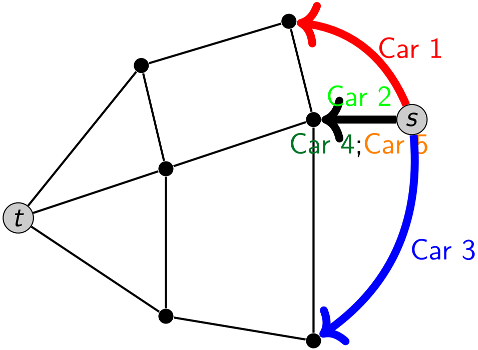

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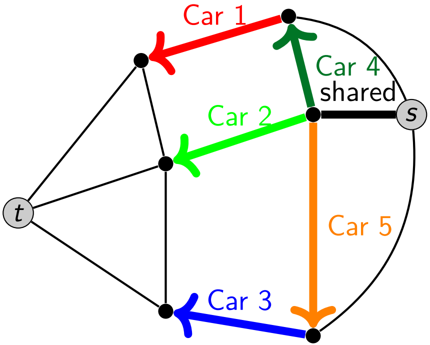

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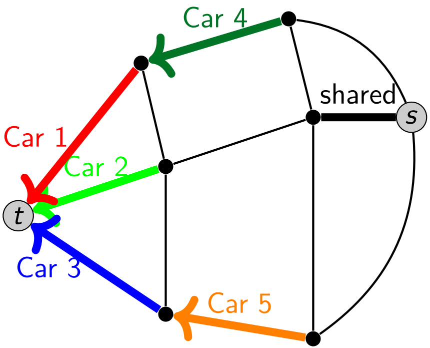

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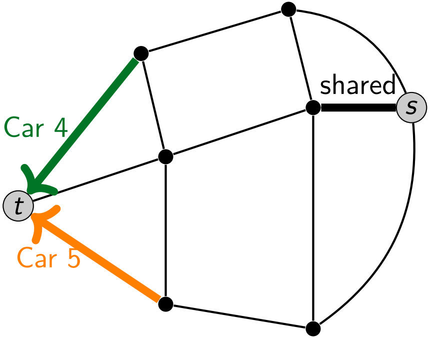

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3. **Dynamic flow** [Ford and Fulkerson: PUP, 1962]

Overview

	Undirected, with k		Directed, with k		DAGs, with k	
	const.	unbound	const.	unbound	const.	unbound
Path-FRCA						
Trail-FRCA						
Walk-RCA						
Walk-FRCA						

$k = \#(\text{shared edges})$

Overview

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Trail-FRCA	NP-c, $k \geq 0$	NP-c, $\Delta \geq 5$	NP-c, $k \geq 0$	NP-c, $\Delta \geq 3$		
Walk-RCA						
Walk-FRCA						

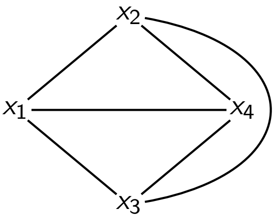
Δ =max Deg. k =#(shared edges)

Planar Hamiltonian Cycle

Planar Hamiltonian Cycle (PHC)

Given: An undirected, planar, and cubic graph $G = (V, E)$.
Question: Is there a cycle in G visiting each vertex in V exactly once?

NP-complete [Garey et al.: SICOMP, 1976]



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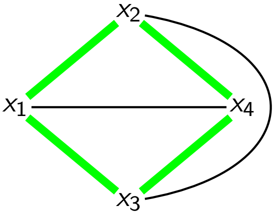
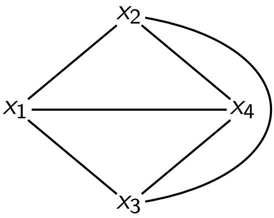


Reduction from **PHC** to **Path-RCA** with $k = 0$



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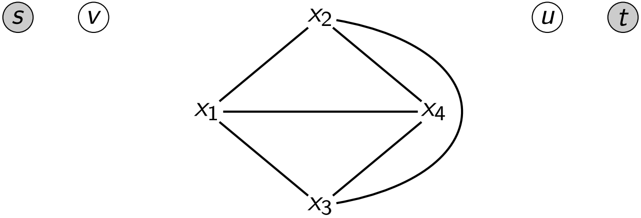

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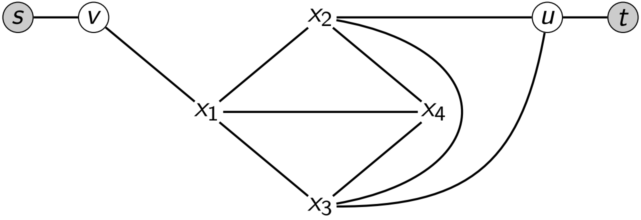

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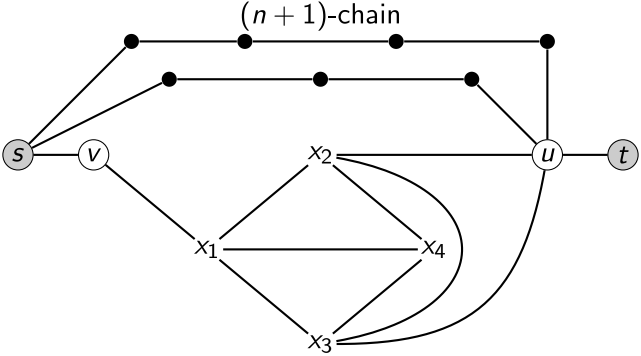

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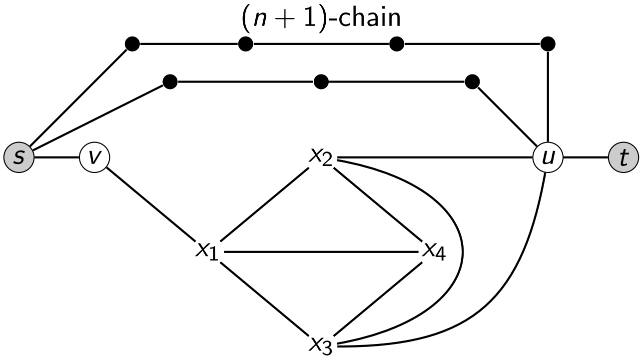
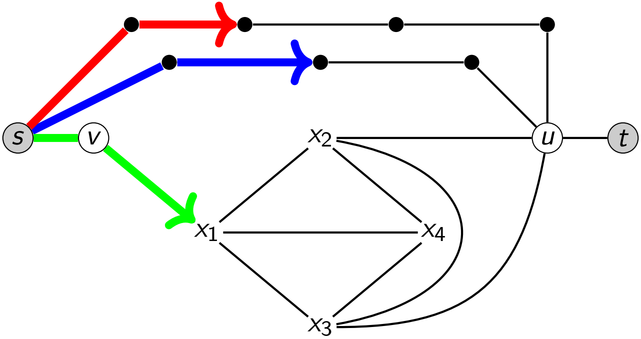


- \# of paths $p = n - 1 = 3$
- \# of shared edges $k = 0$

Construct 3 s-t Paths



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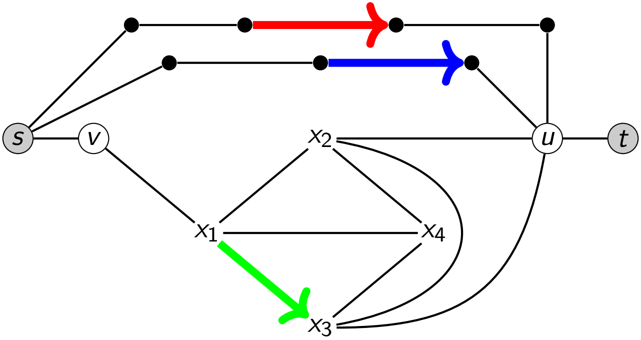

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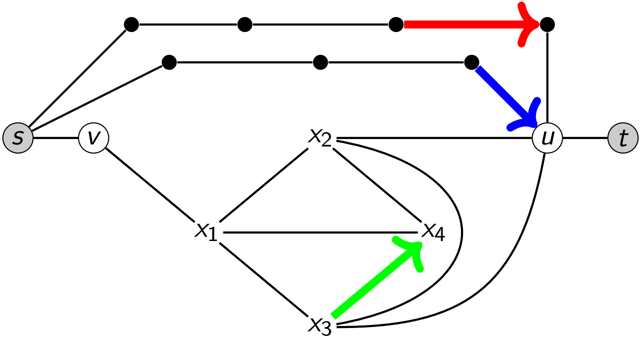

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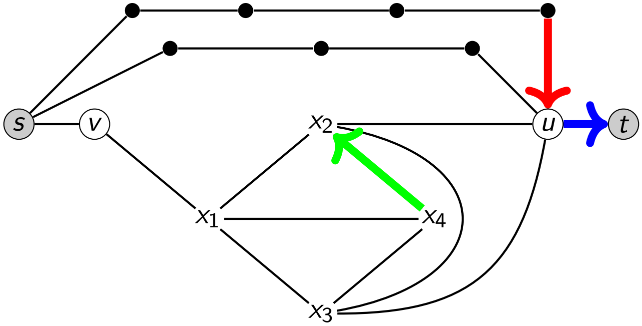

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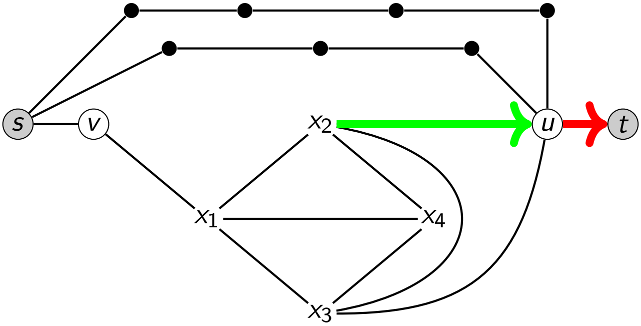

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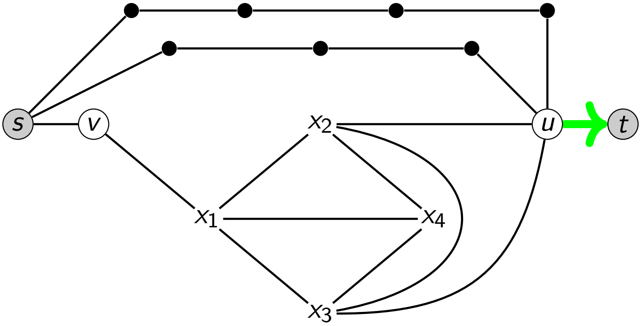

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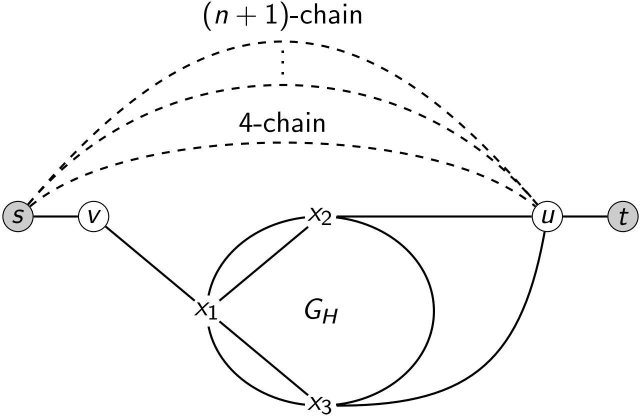

- # of paths $p = n - 1$
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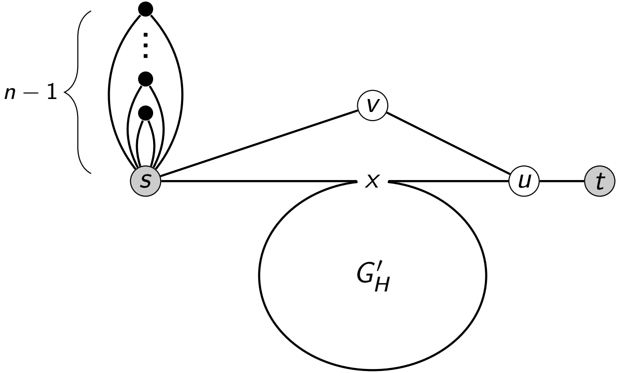
Gadget for Trails



- # of paths $p = 2n$
- # of shared edges $k = 0$

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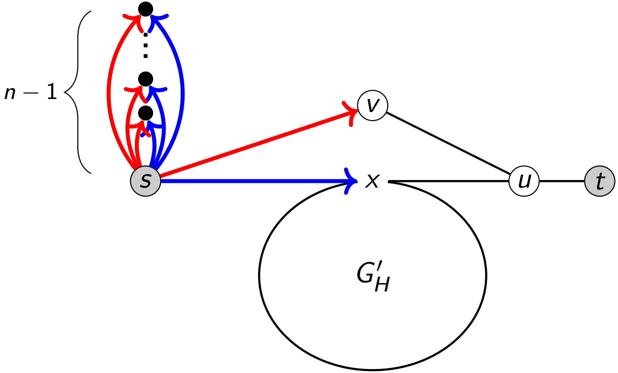

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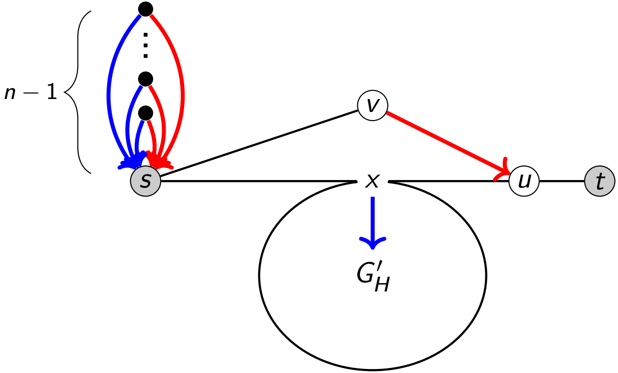

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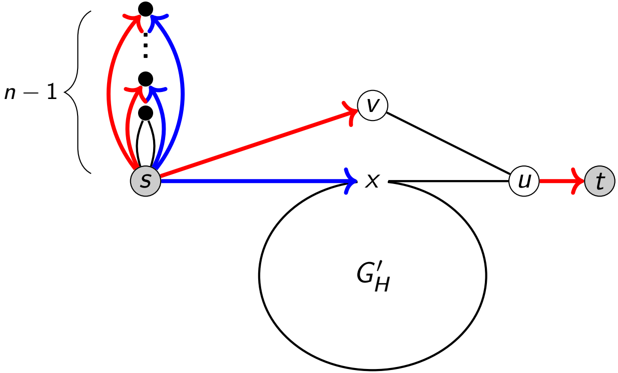

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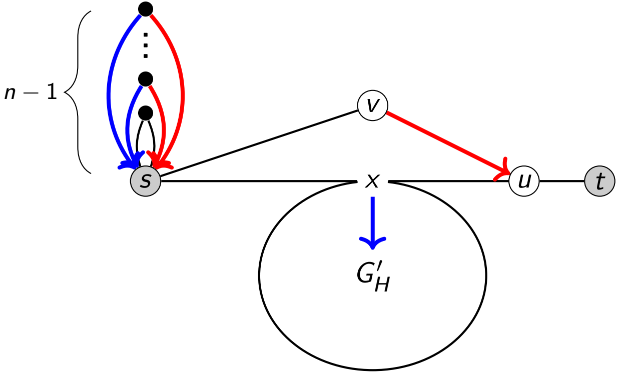

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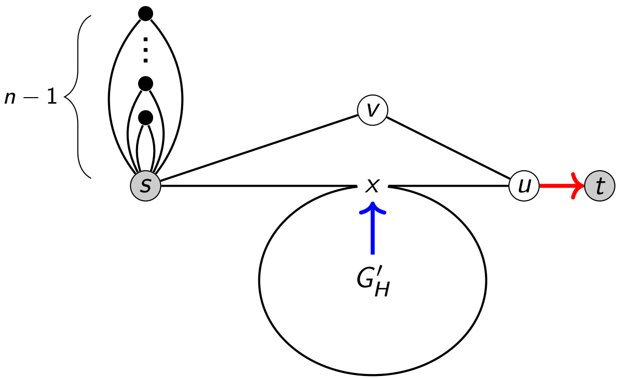

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Polynomial Results on DAGs

	Undirected, with k		Directed, with k		DAGs, with k	
	const.	unbound	const.	unbound	const.	unbound
Path-FRCA	NP-c, $k \geq 0$	NP-c, $\Delta \geq 4$	NP-c, $k \geq 0$	NP-c, $\Delta \geq 4$	**P**	
Trail-FRCA	NP-c, $k \geq 0$	NP-c, $\Delta \geq 5$	NP-c, $k \geq 0$	NP-c, $\Delta \geq 3$	**P**	
Walk-RCA					**P**	
Walk-FRCA					**P**	

(F)RCA on Directed Acyclic Graphs for const. k

In acyclic graphs, no vertex can appear twice in an s-t-route

$\implies$ *Walk* $=$ *Trail* $=$ *Path*

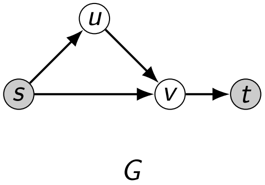
(F)RCA on Directed Acyclic Graphs for const. k

In acyclic graphs, no vertex can appear twice in an s-t-route

$\implies$ $Walk = Trail = Path$

In directed graphs, arcs can only be shared in one direction

(F)RCA on Directed Acyclic Graphs for const. k



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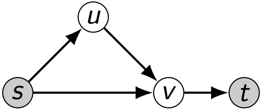
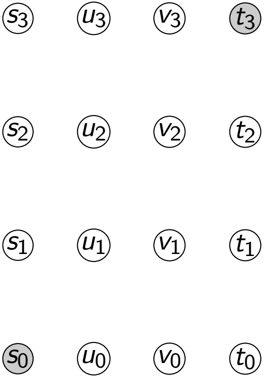


G



G'

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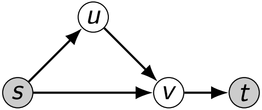

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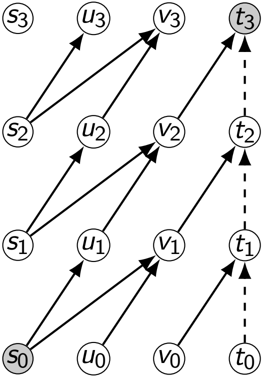

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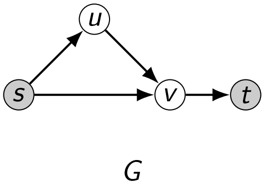

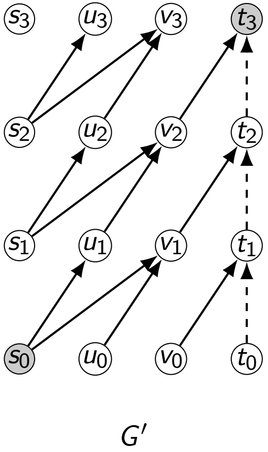

Maximum flow in G': $\mathcal{O}(n'm')$

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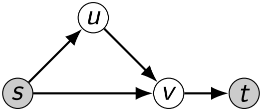

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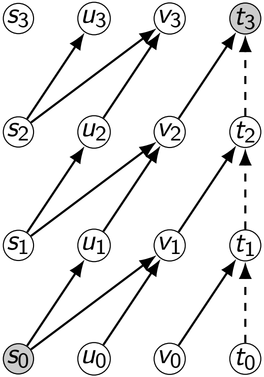

G'

Running time in G: $\mathcal{O}(n^3 m \cdot m^k)$

Results

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Path-FRCA	NP-c, $k \geq 0$	NP-c, $\Delta \geq 4$	NP-c, $k \geq 0$	NP-c, $\Delta \geq 4$	P	
Trail-FRCA	NP-c, $k \geq 0$	NP-c, $\Delta \geq 5$	NP-c, $k \geq 0$	NP-c, $\Delta \geq 3$	P	
Walk-RCA					P	
Walk-FRCA					P	

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Walk-FRCA			P		P	

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Walk-RCA	P	P	P		P	
Walk-FRCA			P		P	

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Walk-RCA	P	P	P	NP-c, W[2]-hard	P	NP-c, W[2]-hard
Walk-FRCA		NP-c, W[2]-hard	P	NP-c, W[2]-hard	P	NP-c, W[2]-hard

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Path-FRCA	NP-c, $k \geq 0$	NP-c, $\Delta \geq 4$	NP-c, $k \geq 0$	NP-c, $\Delta \geq 4$	P	NP-c, W[2]-hard
Trail-FRCA	NP-c, $k \geq 0$	NP-c, $\Delta \geq 5$	NP-c, $k \geq 0$	NP-c, $\Delta \geq 3$	P	NP-c, W[2]-hard
Walk-RCA	P	P	P	NP-c, W[2]-hard	P	NP-c, W[2]-hard
Walk-FRCA	open	NP-c, W[2]-hard	P	NP-c, W[2]-hard	P	NP-c, W[2]-hard

Challenges for Future Research

	Undirected, with k		Directed, with k		DAGs, with k	
	const.	unbound	const.	unbound	const.	unbound
Path-FRCA	NP-c, $k \geq 0$	NP-c, $\Delta \geq 4$	NP-c, $k \geq 0$	NP-c, $\Delta \geq 4$	P	NP-c, W[2]-hard
Trail-FRCA	NP-c, $k \geq 0$	NP-c, $\Delta \geq 5$	NP-c, $k \geq 0$	NP-c, $\Delta \geq 3$	P	NP-c, W[2]-hard
Walk-RCA	P	P	P	NP-c, W[2]-hard	P	NP-c, W[2]-hard
Walk-FRCA	open	NP-c, W[2]-hard	P	NP-c, W[2]-hard	P	NP-c, W[2]-hard

- **Walk-FRCA** NP-complete for constant k?

Challenges for Future Research

	Undirected, with k		Directed, with k		DAGs, with k	
	const.	unbound	const.	unbound	const.	unbound
Path-FRCA	NP-c, $k \geq 0$	NP-c, $\Delta \geq 4$	NP-c, $k \geq 0$	NP-c, $\Delta \geq 4$	P	NP-c, W[2]-hard
Trail-FRCA	NP-c, $k \geq 0$	NP-c, $\Delta \geq 5$	NP-c, $k \geq 0$	NP-c, $\Delta \geq 3$	P	NP-c, W[2]-hard
Walk-RCA	P	P	P	NP-c, W[2]-hard	P	NP-c, W[2]-hard
Walk-FRCA	open	NP-c, W[2]-hard	P	NP-c, W[2]-hard	P	NP-c, W[2]-hard

- **Walk-FRCA** NP-complete for constant k?
- Complexity when restricting the number of routes p

Challenges for Future Research

	Undirected, with k const.	Undirected, with k unbound	Directed, with k const.	Directed, with k unbound	DAGs, with k const.	DAGs, with k unbound
Path-FRCA	NP-c, $k \geq 0$	NP-c, $\Delta \geq 4$	NP-c, $k \geq 0$	NP-c, $\Delta \geq 4$	P	NP-c, W[2]-hard
Trail-FRCA	NP-c, $k \geq 0$	NP-c, $\Delta \geq 5$	NP-c, $k \geq 0$	NP-c, $\Delta \geq 3$	P	NP-c, W[2]-hard
Walk-RCA	P	P	P	NP-c, W[2]-hard	P	NP-c, W[2]-hard
Walk-FRCA	open	NP-c, W[2]-hard	P	NP-c, W[2]-hard	P	NP-c, W[2]-hard

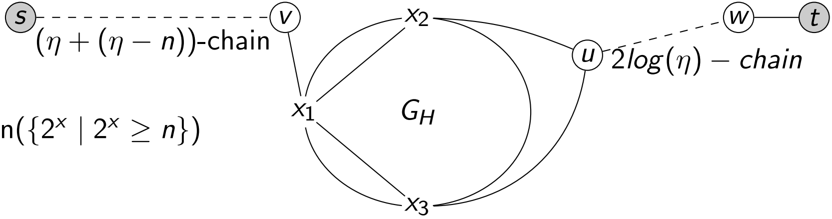
- **Walk-FRCA** NP-complete for constant k?
- Complexity when restricting the number of routes p
- FPT for combined parameter $(\Delta + k)$

Challenges for Future Research

	Undirected, with k		Directed, with k		DAGs, with k	
	const.	unbound	const.	unbound	const.	unbound
Path-FRCA	NP-c, $k \geq 0$	NP-c, $\Delta \geq 4$	NP-c, $k \geq 0$	NP-c, $\Delta \geq 4$	P	NP-c, W[2]-hard
Trail-FRCA	NP-c, $k \geq 0$	NP-c, $\Delta \geq 5$	NP-c, $k \geq 0$	NP-c, $\Delta \geq 3$	P	NP-c, W[2]-hard
Walk-RCA	P	P	P	NP-c, W[2]-hard	P	NP-c, W[2]-hard
Walk-FRCA	open	NP-c, W[2]-hard	P	NP-c, W[2]-hard	P	NP-c, W[2]-hard

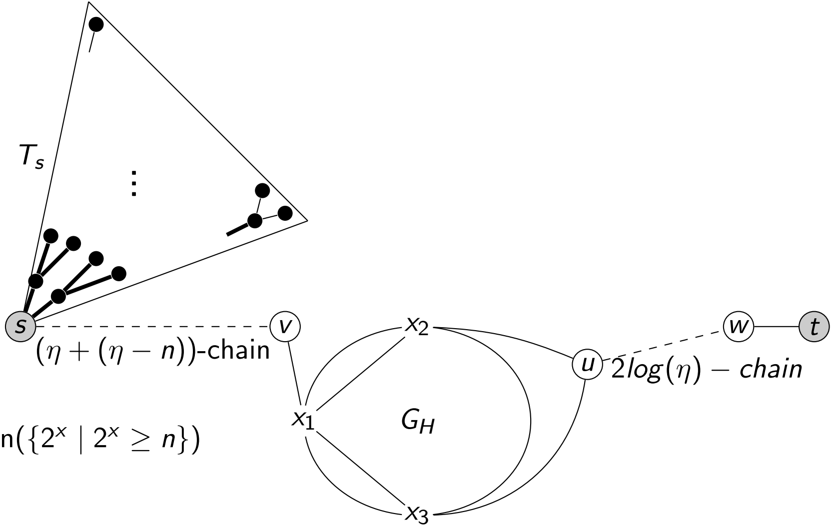
Thank You!

Gadget for $\Delta \geq 4$



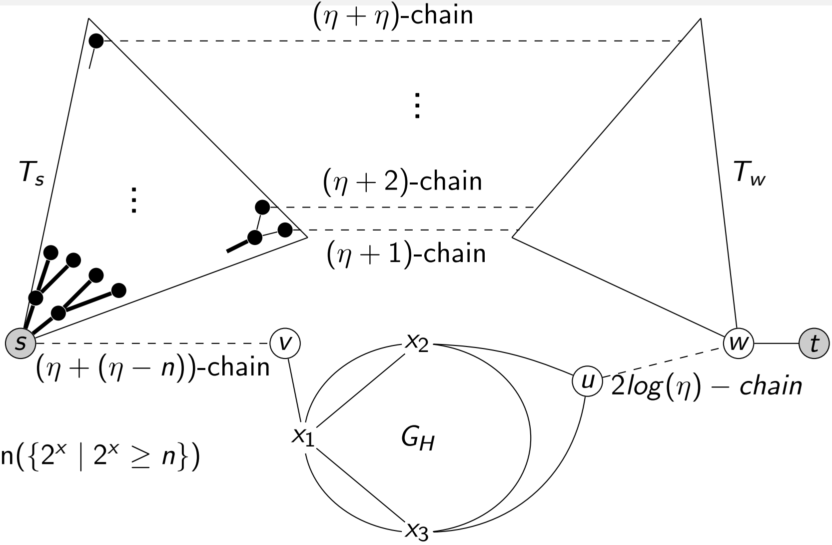
$$\eta = \min(\{2^x \mid 2^x \geq n\})$$

Gadget for $\Delta \geq 4$



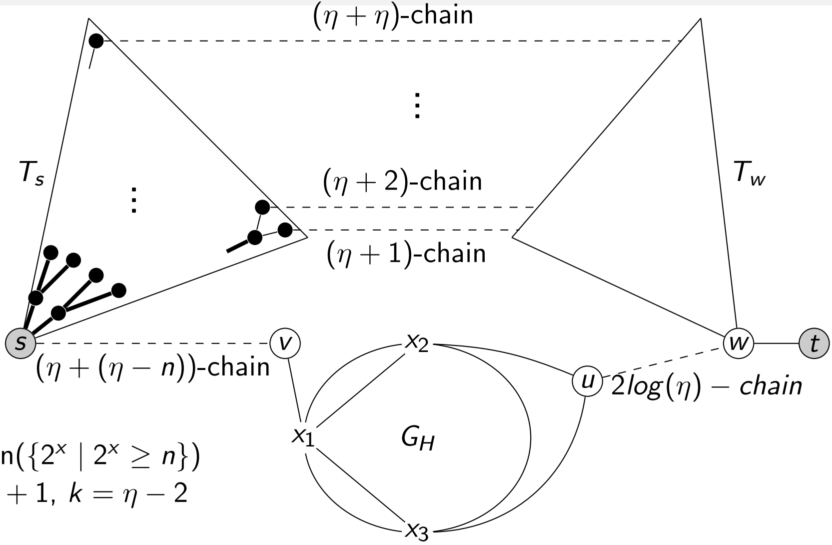
$\eta = \min(\{2^x \mid 2^x \geq n\})$

Gadget for $\Delta \geq 4$



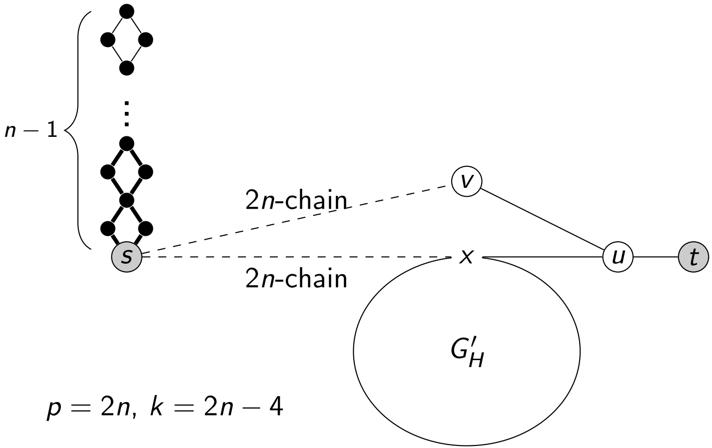
$\eta = \min(\{2^x \mid 2^x \geq n\})$

Gadget for $\Delta \geq 4$



$$\eta = \min(\{2^x \mid 2^x \geq n\})$$
$$p = \eta + 1,\ k = \eta - 2$$

Gadget for Trails with $\Delta \geq 4$



$p = 2n,\ k = 2n - 4$

Reduction from Setcover

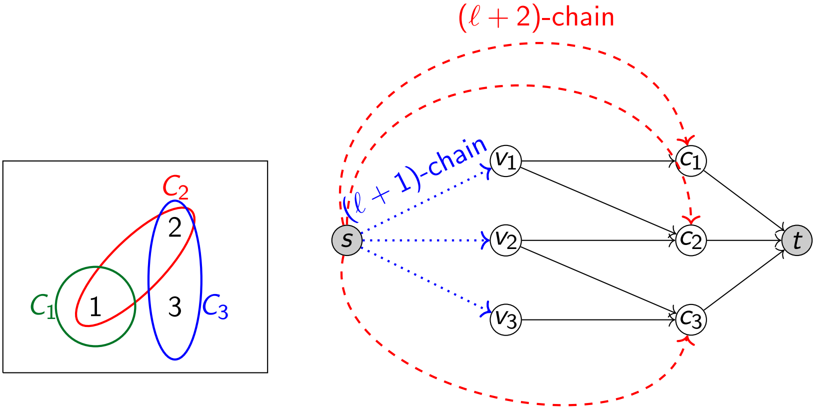


Figure: red dashed line = $(\ell + 2)$-chain; blue dotted line = $(\ell + 1)$-chain.