Polynomial-Time Algorithms for the Subset Feedback Vertex Set Problem on Interval Graphs and Permutation Graphs

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Feedback Vertex Set (FVS)

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Input: A graph $G = (V, E)$

Output: Find a set $X \subset V$ of minimum cardinality such that $G - X$ is acyclic (forest).

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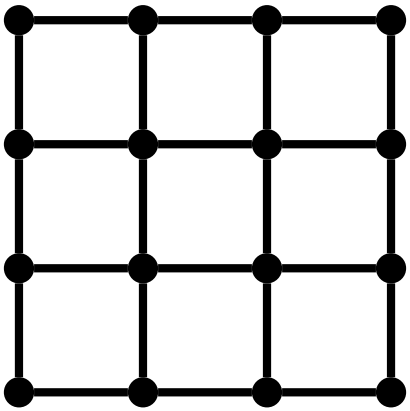

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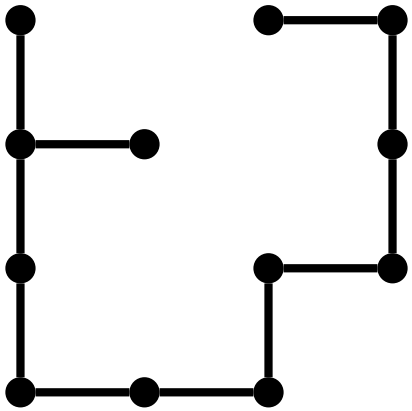

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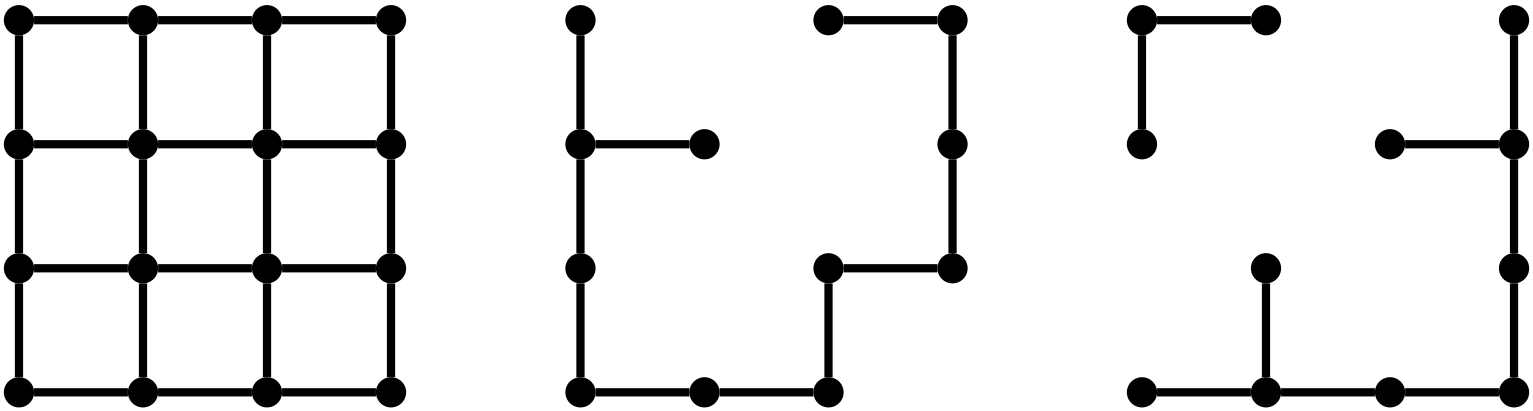

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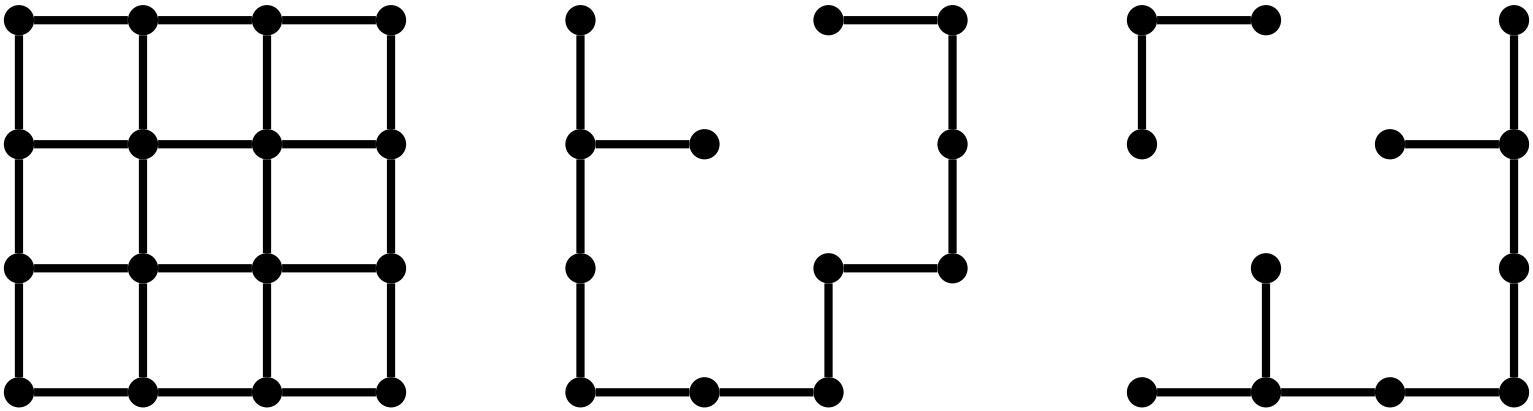

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- Weights on $V \rightarrow$ minimize $\sum_{v \in X} w(v)$

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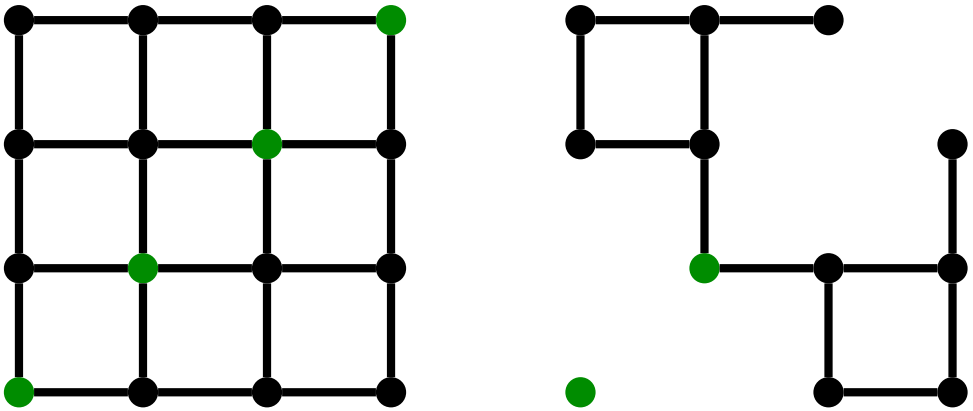

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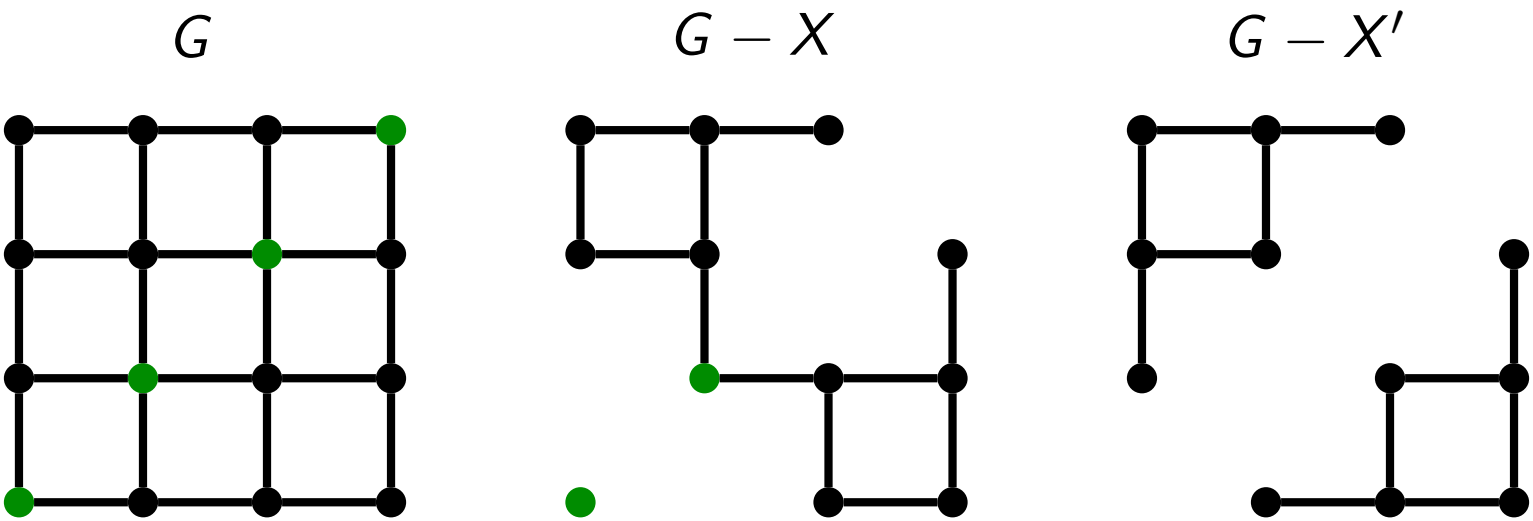

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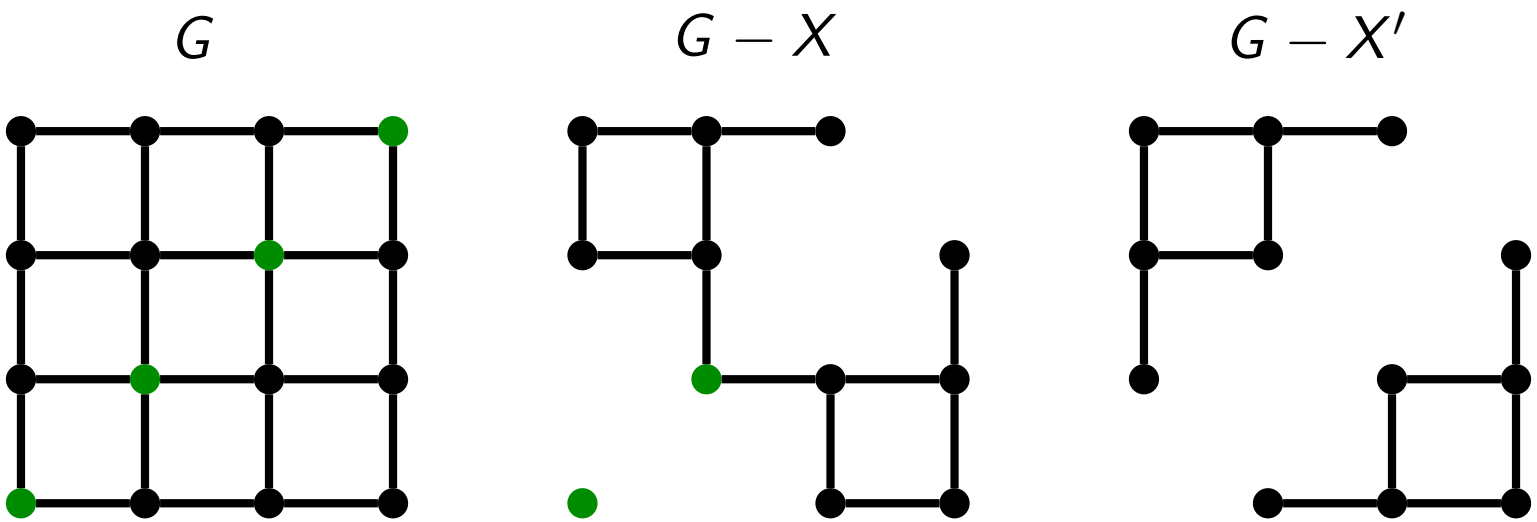

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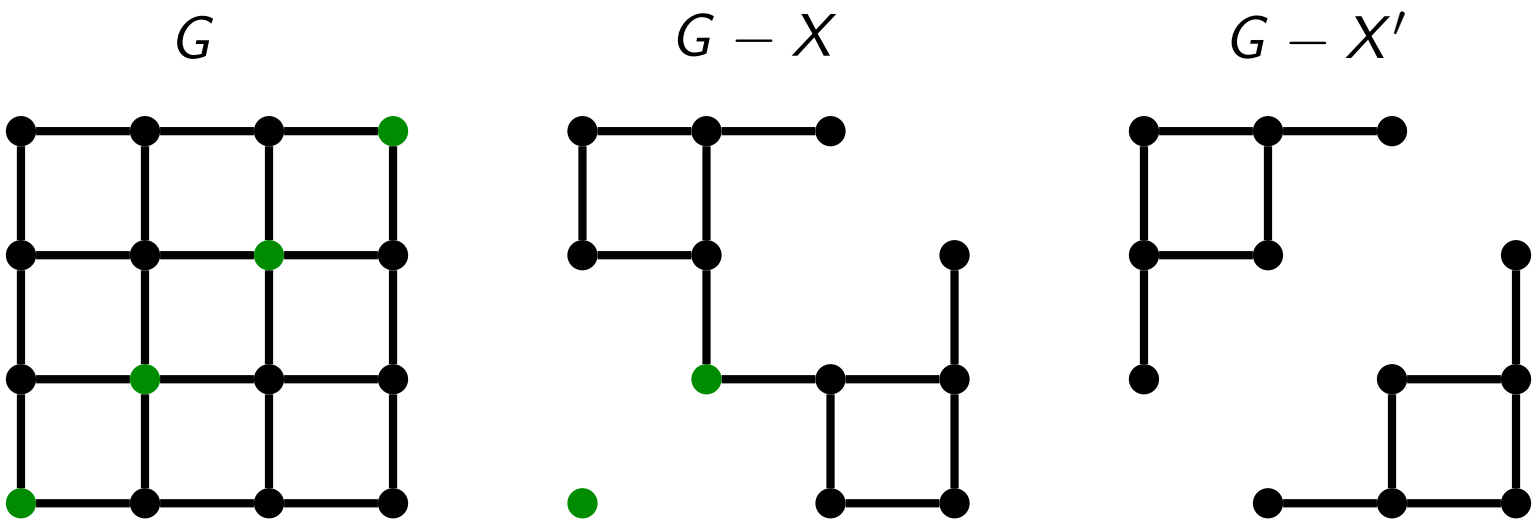

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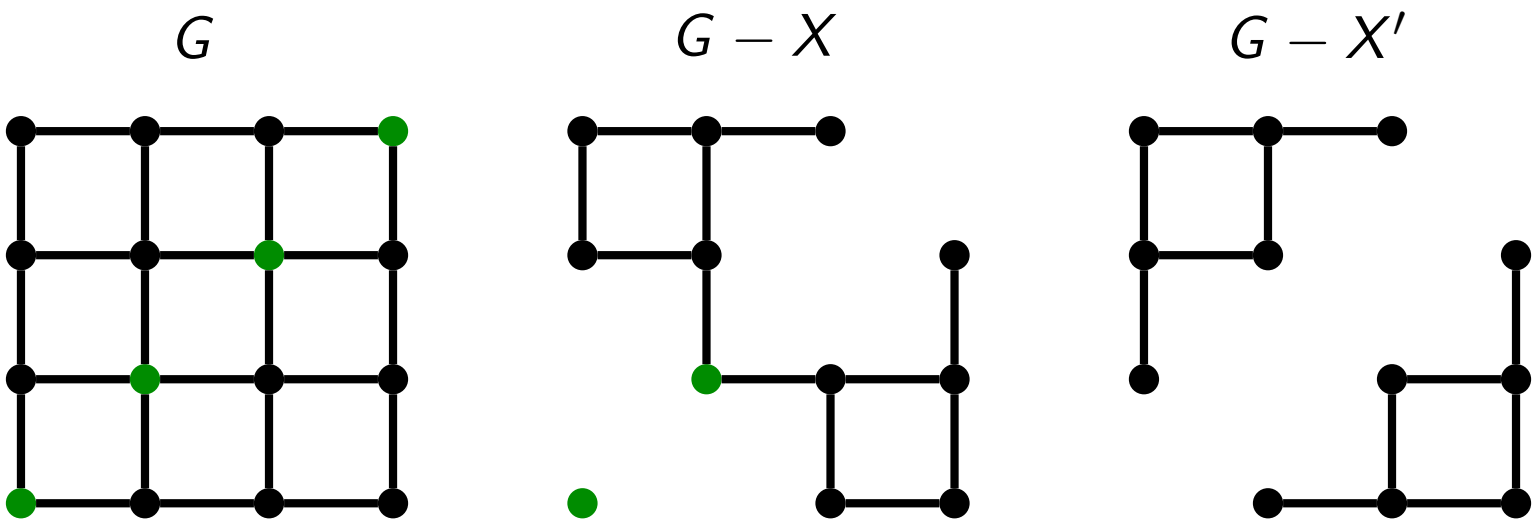

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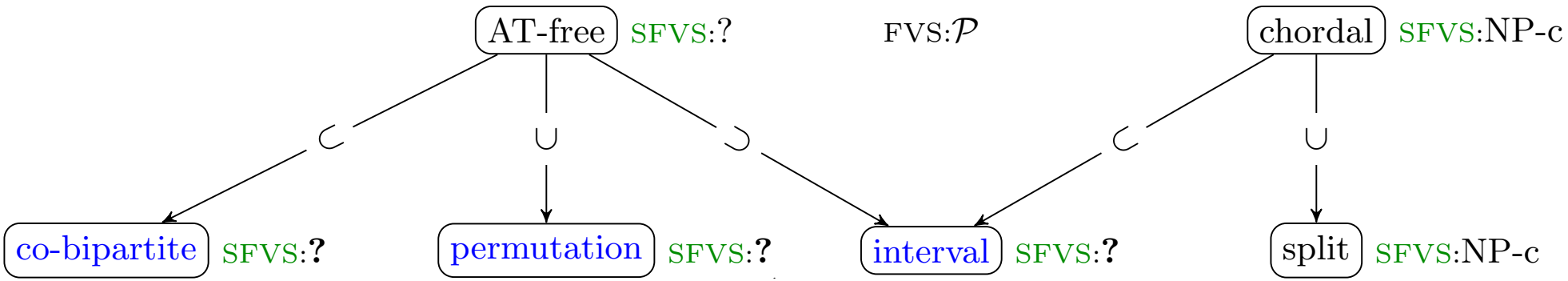

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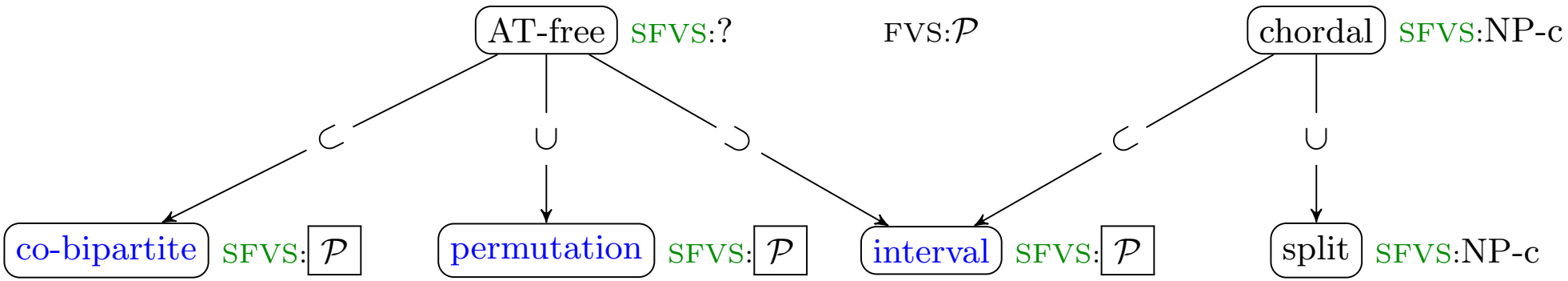

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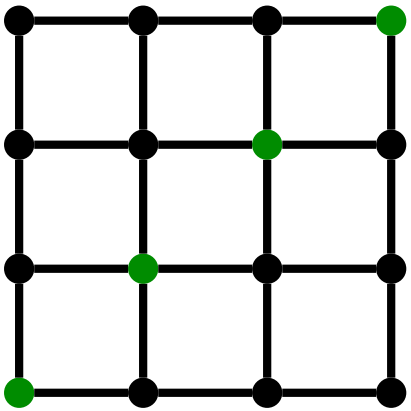

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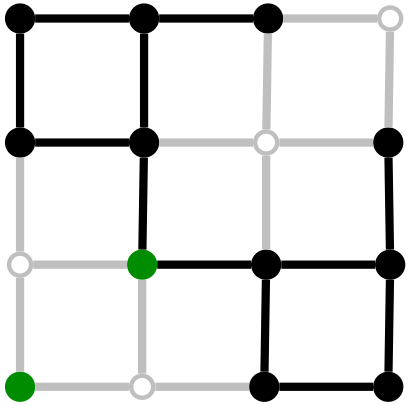

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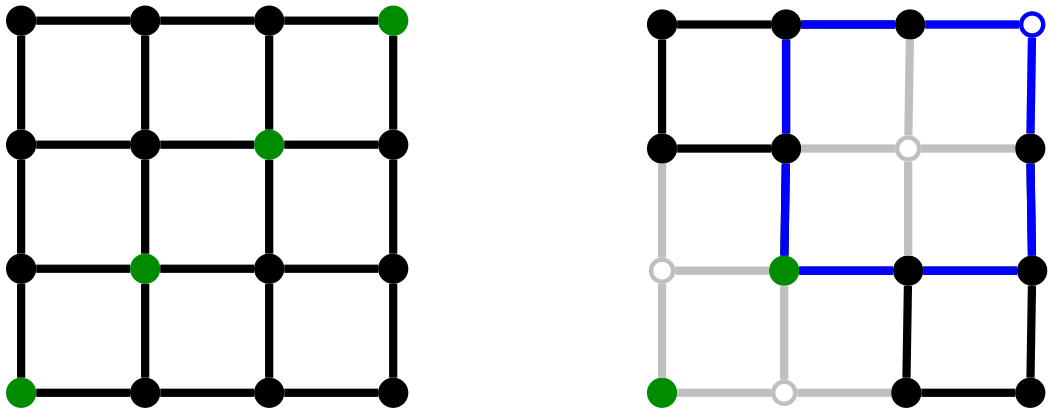

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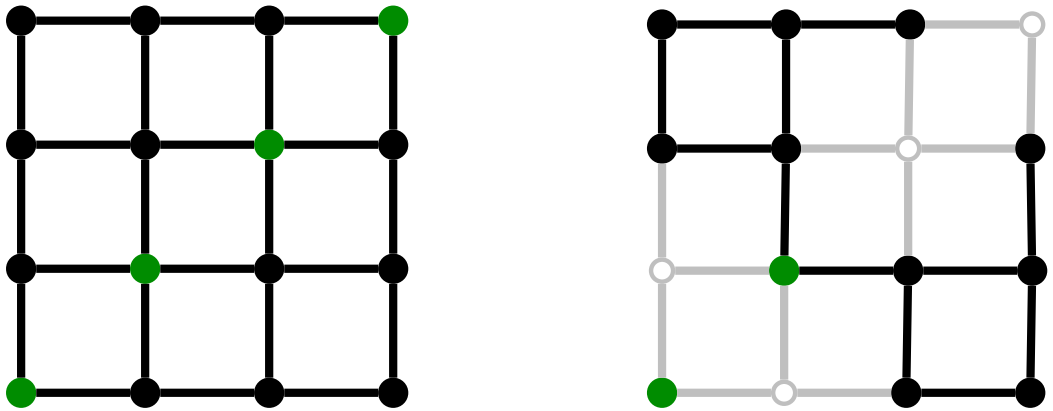

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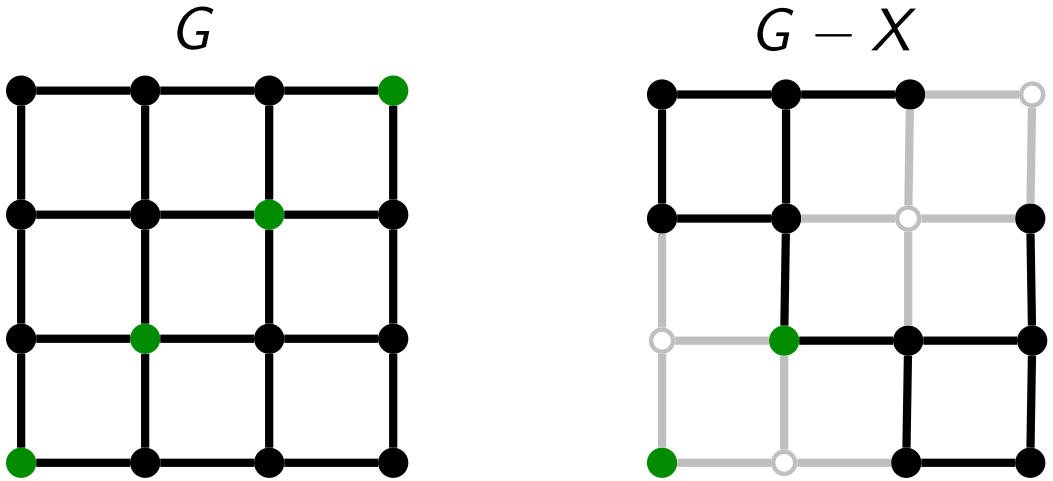

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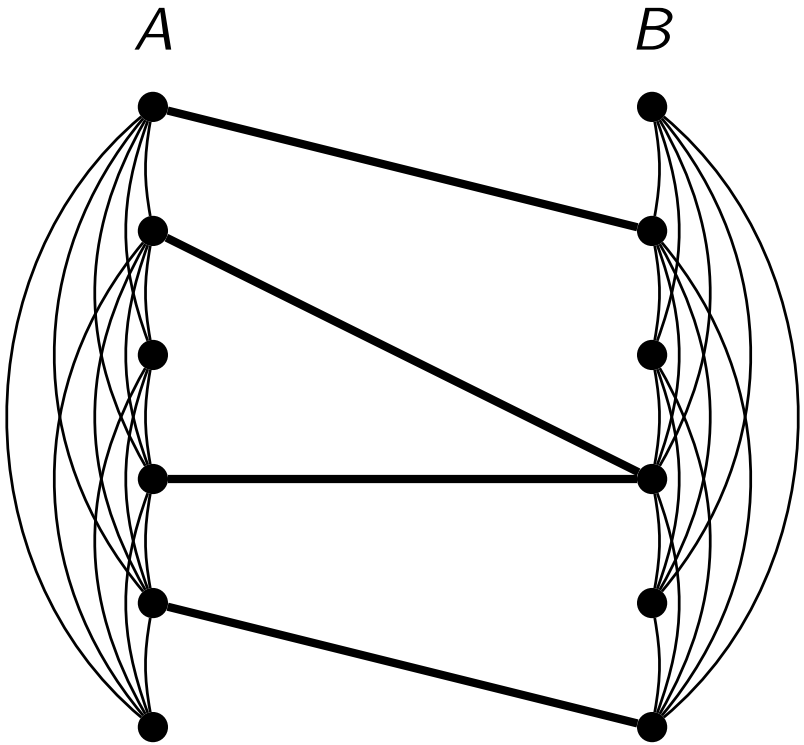
Enumerate all maximal S-forests

Complements of bipartite graphs

- co-bipartite graphs: V is partitioned into two cliques A and B



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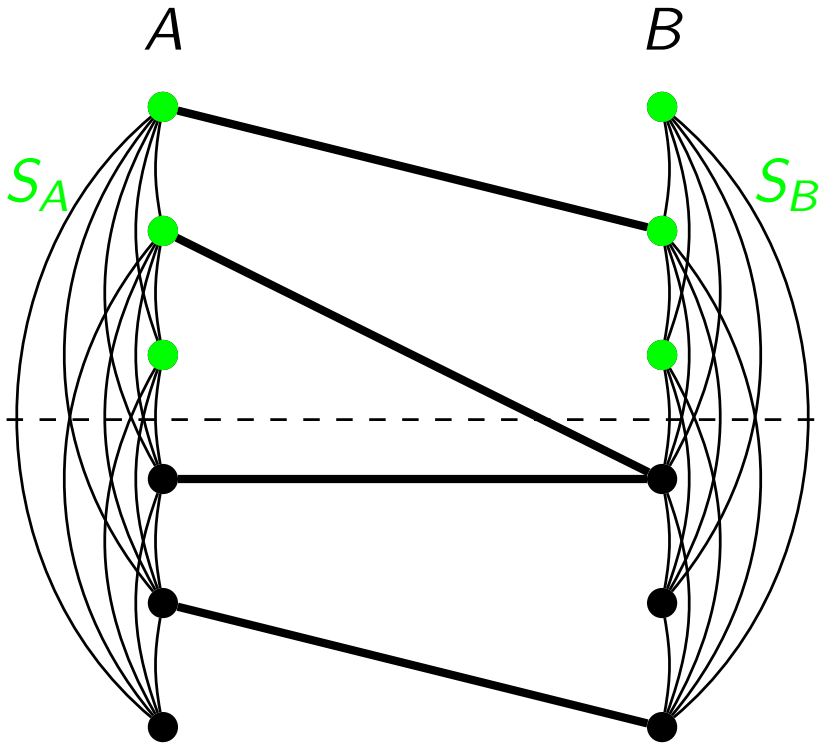

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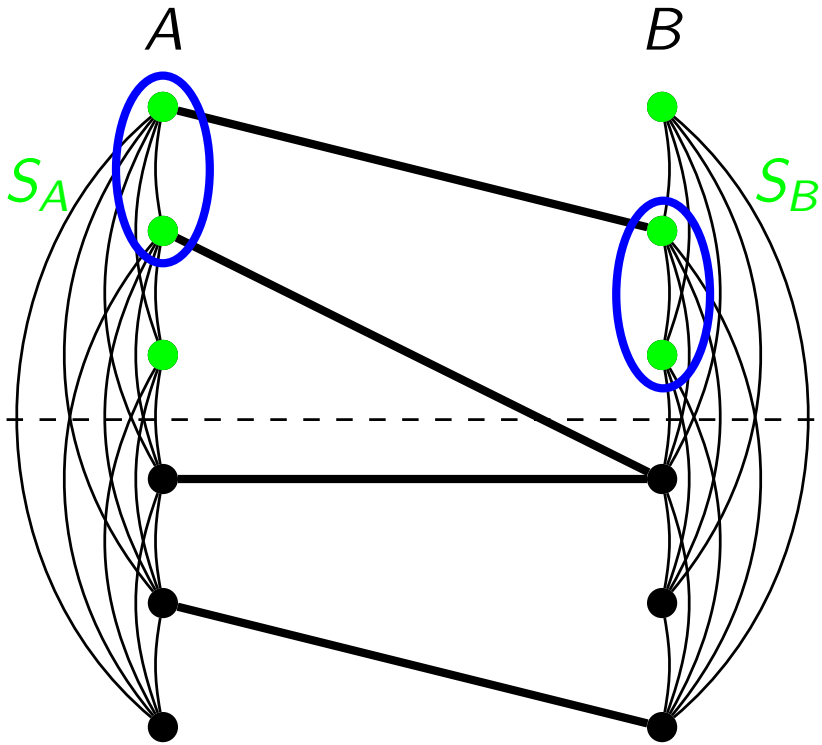

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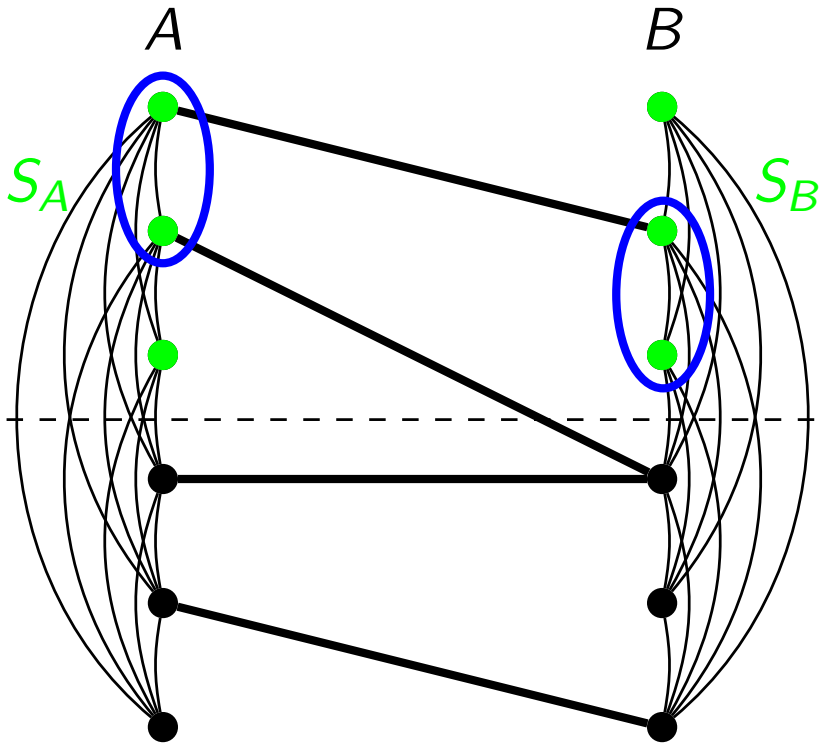

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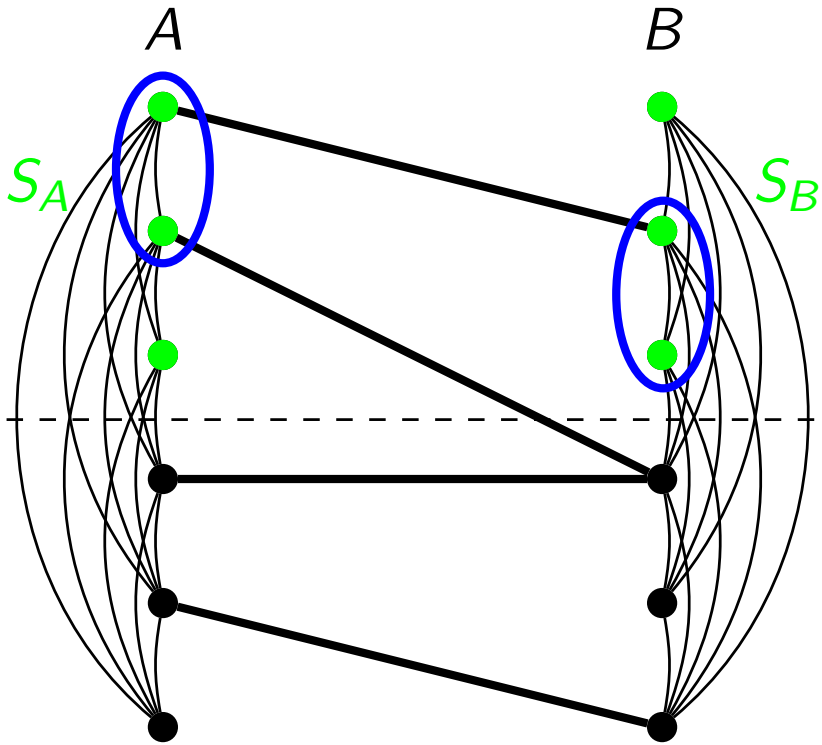

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$\Rightarrow$ An $O(n^4)$ algorithm for computing SFVS on co-biparite graphs.

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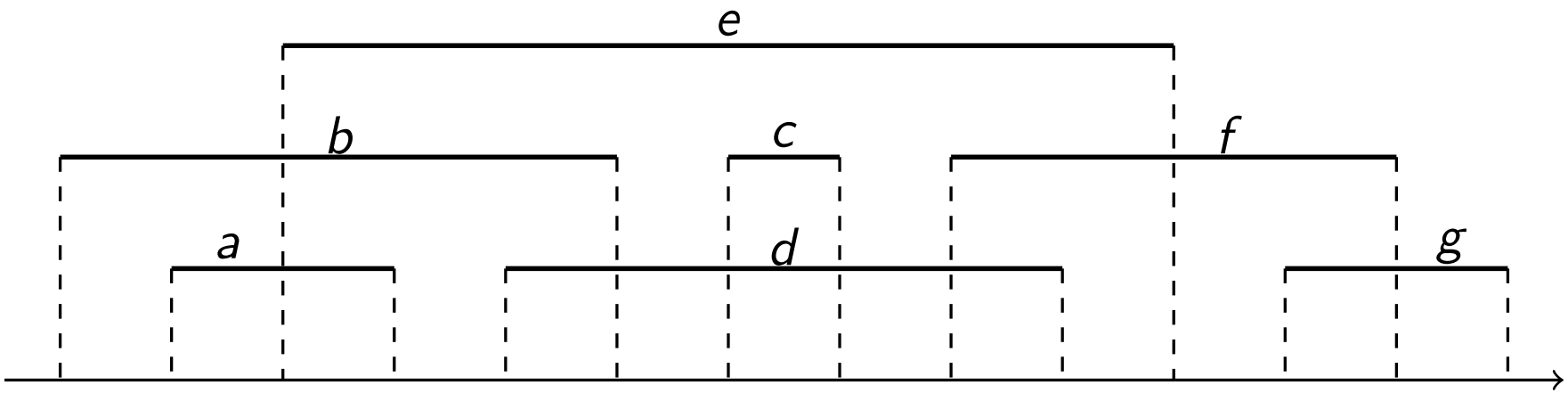

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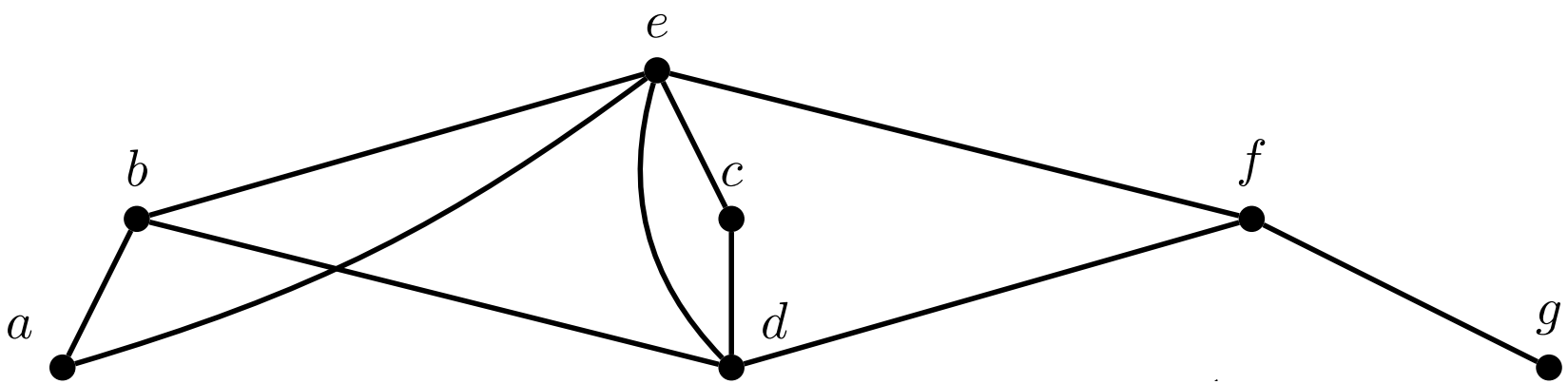

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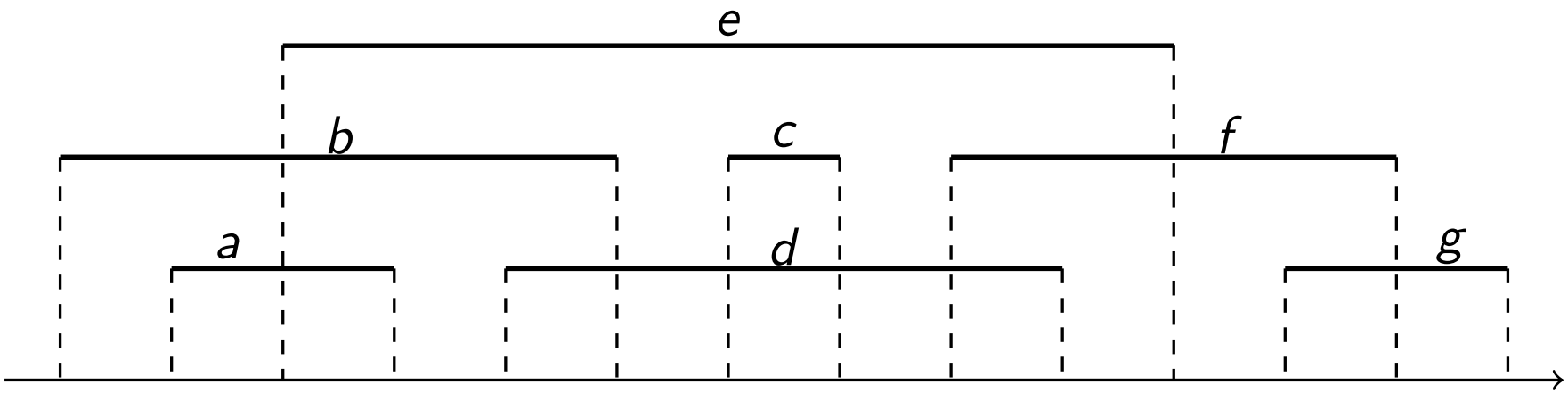

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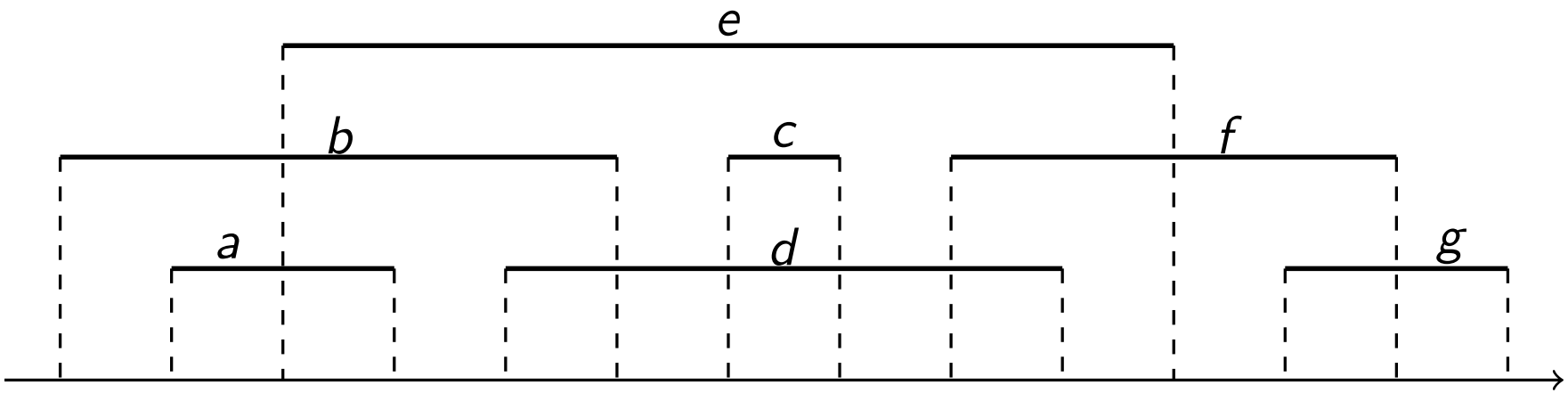

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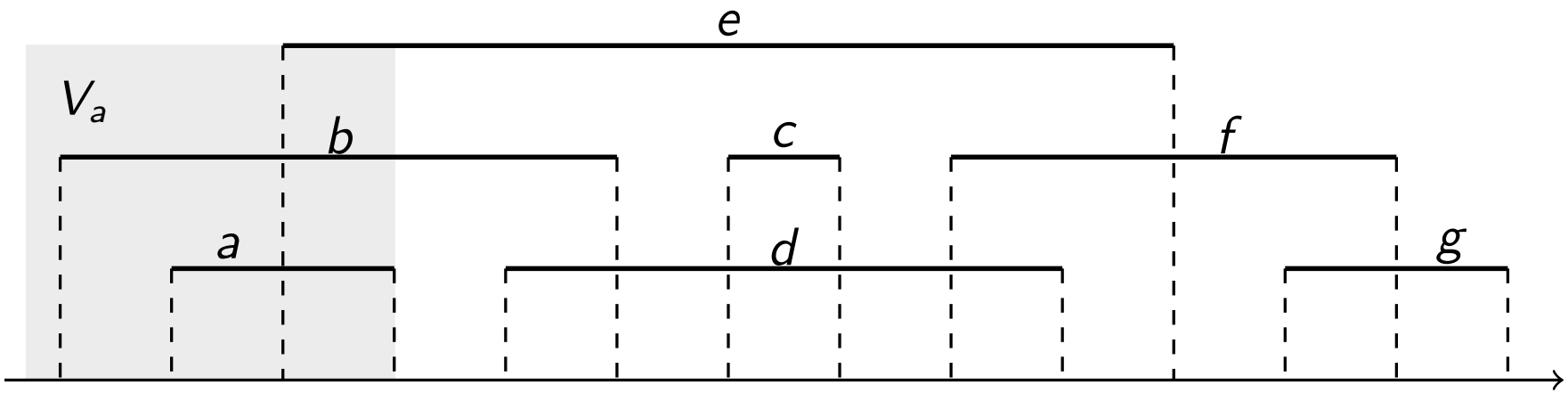

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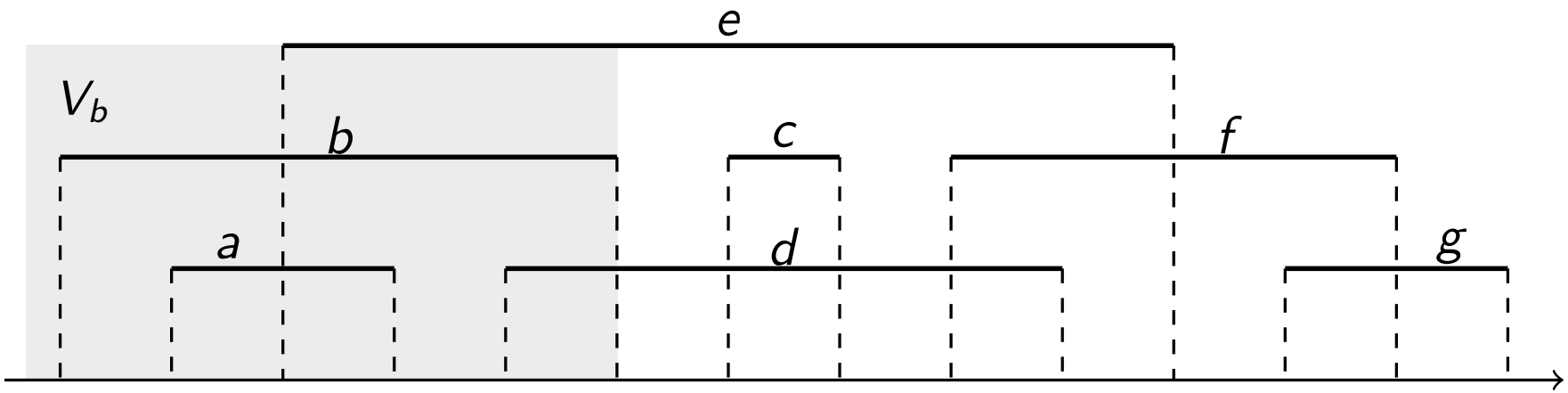

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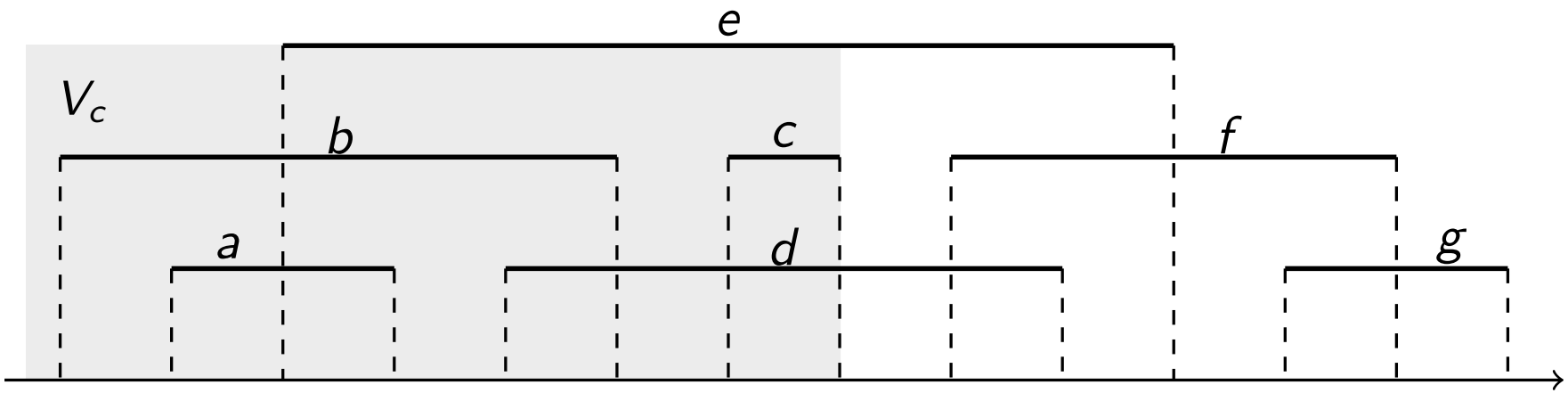

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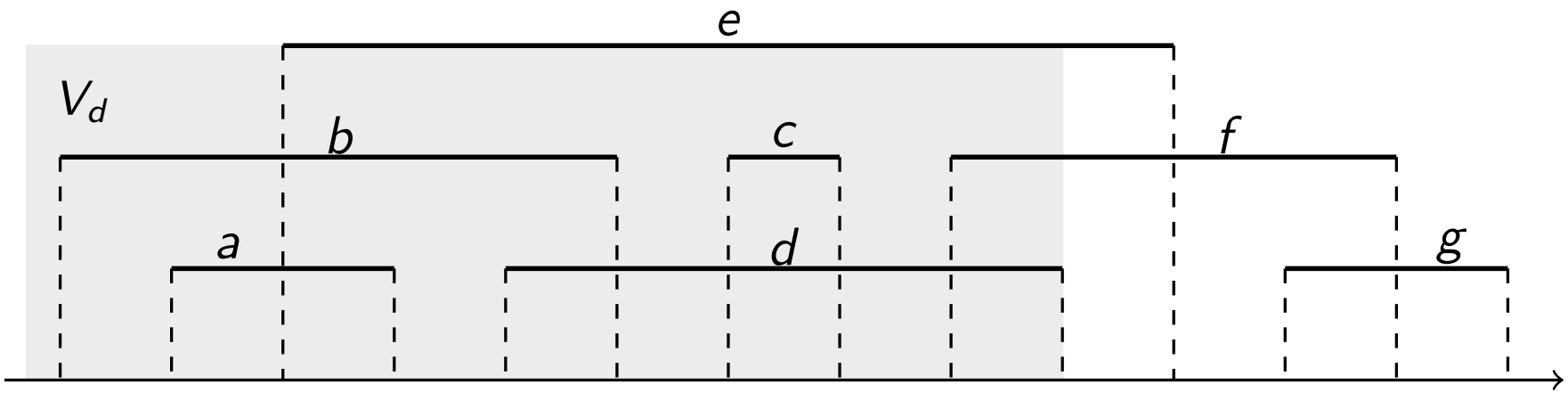

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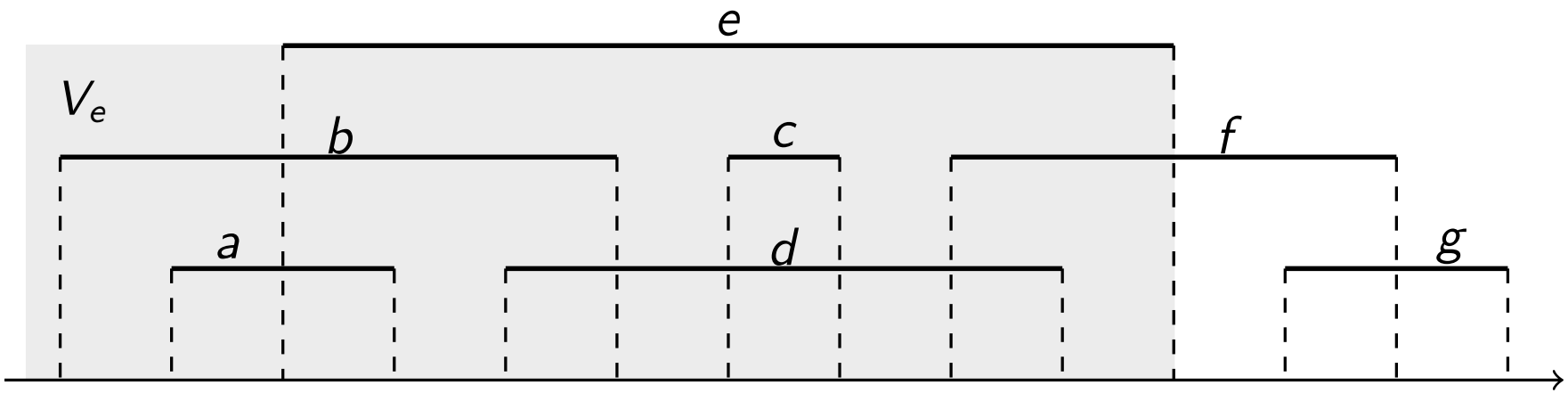

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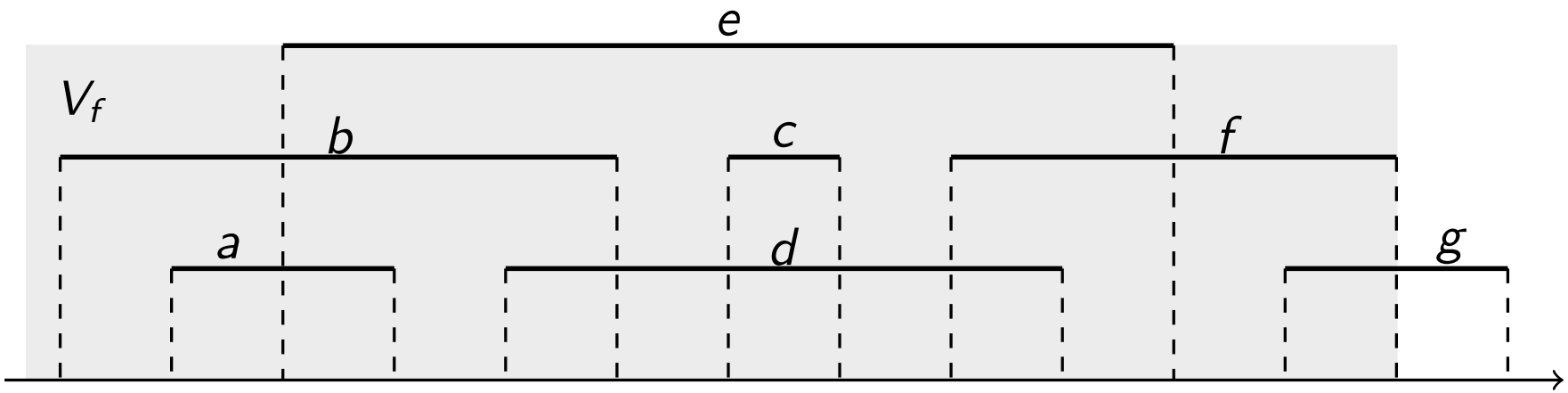

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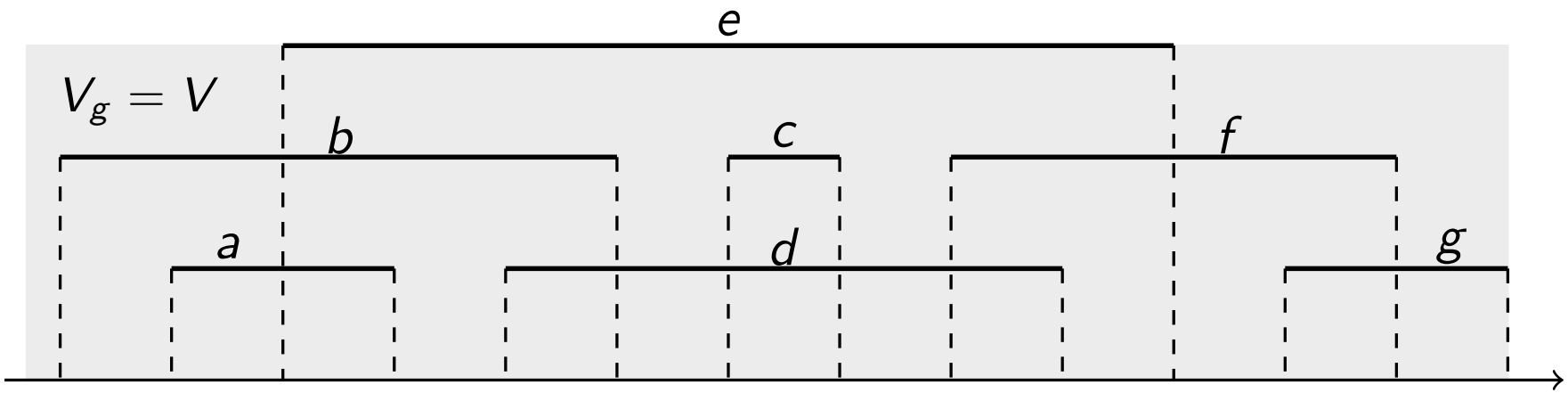

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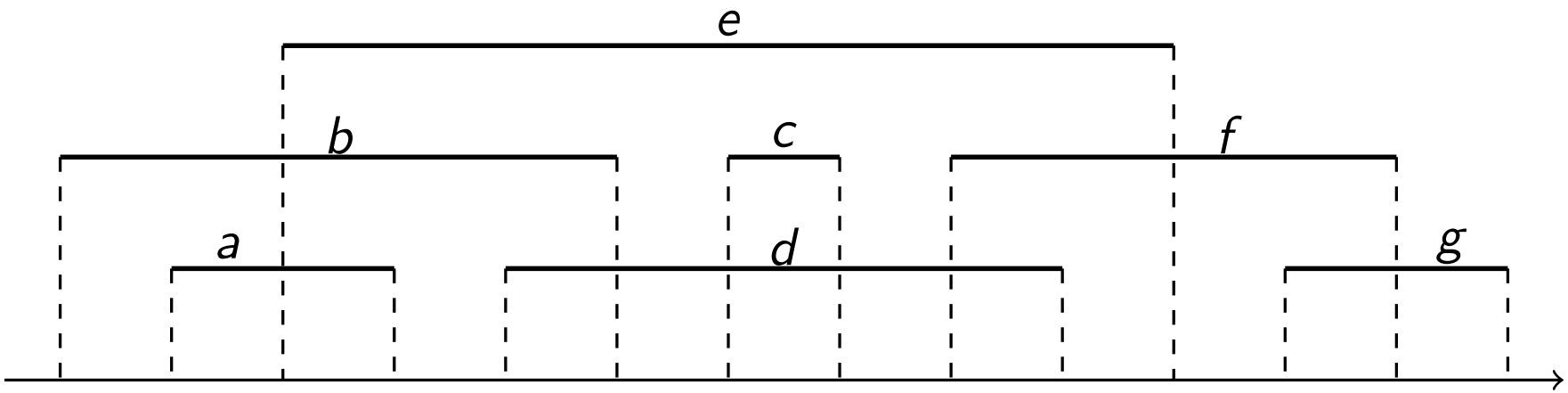
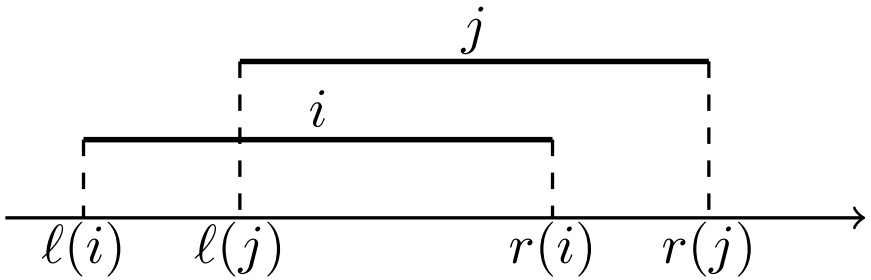
Predecessors



- The left and right endpoints of the intervals define $\leq_\ell$ and $\leq_r$:



$$i \leq_l j \iff \ell(i) \leq \ell(j)$$
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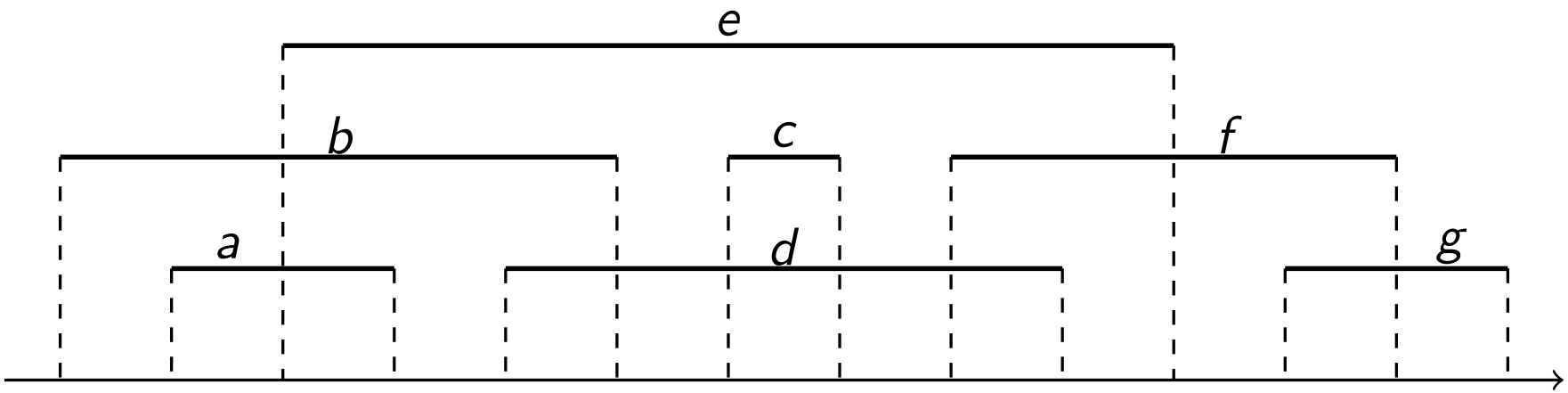

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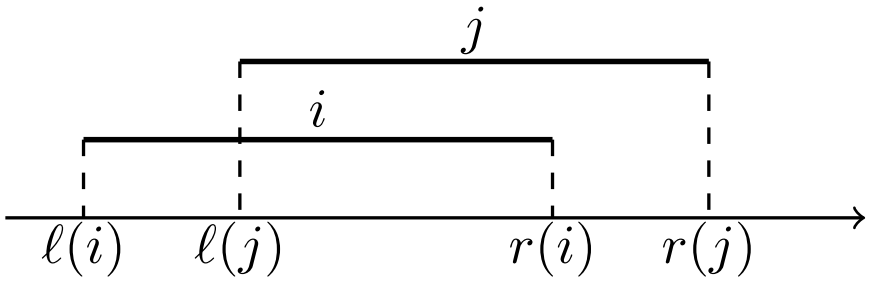

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Predecessors of an interval i:

- $<i = r\text{-}\max(V_i \setminus \{i\})$
  the last interval that finishes before i finishes
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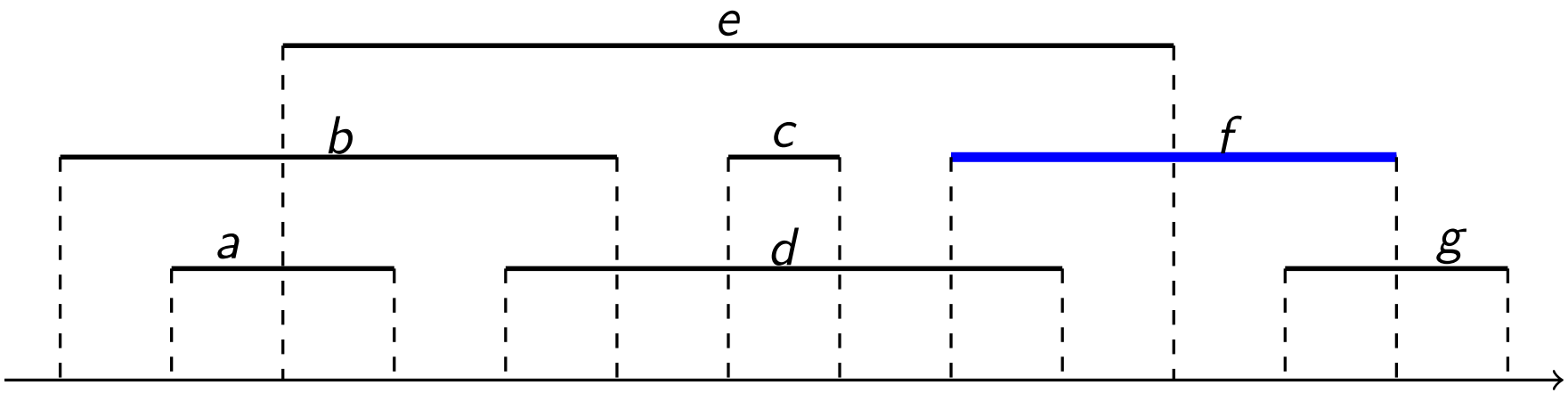
Predecessors



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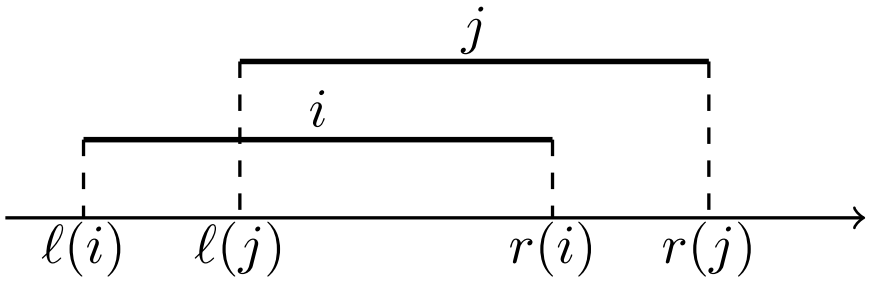

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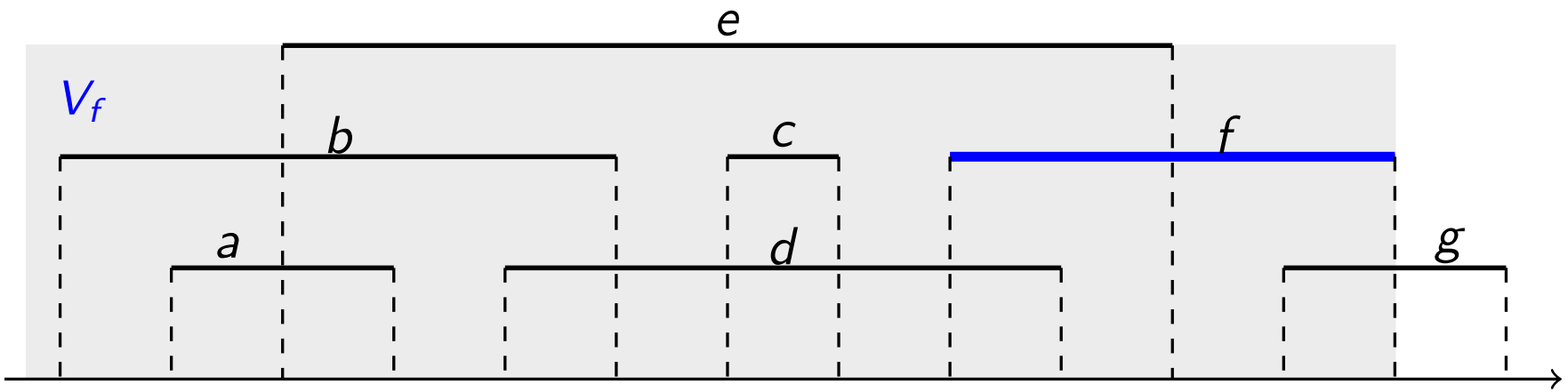

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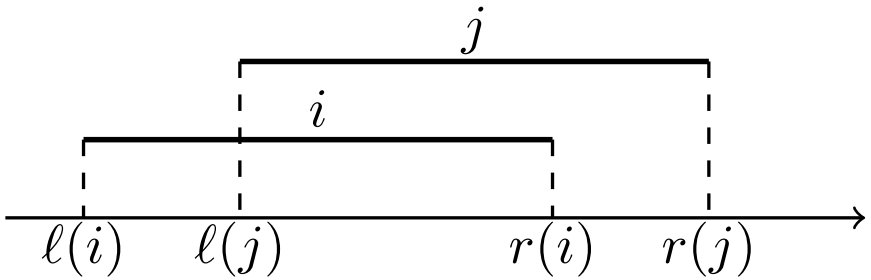

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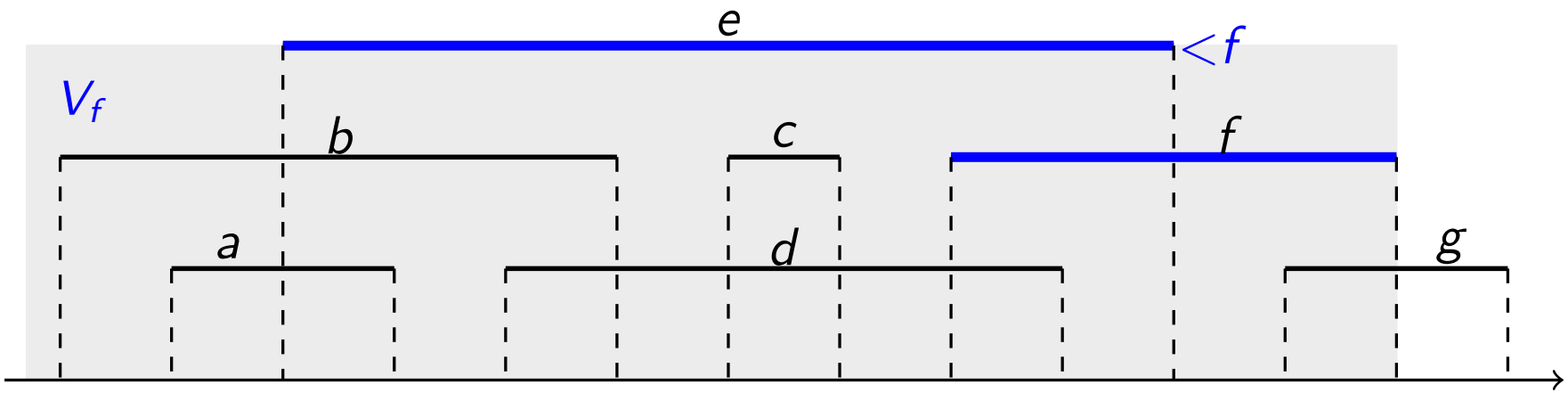

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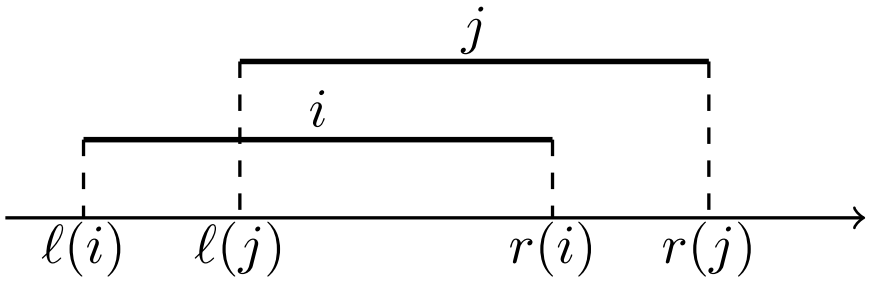

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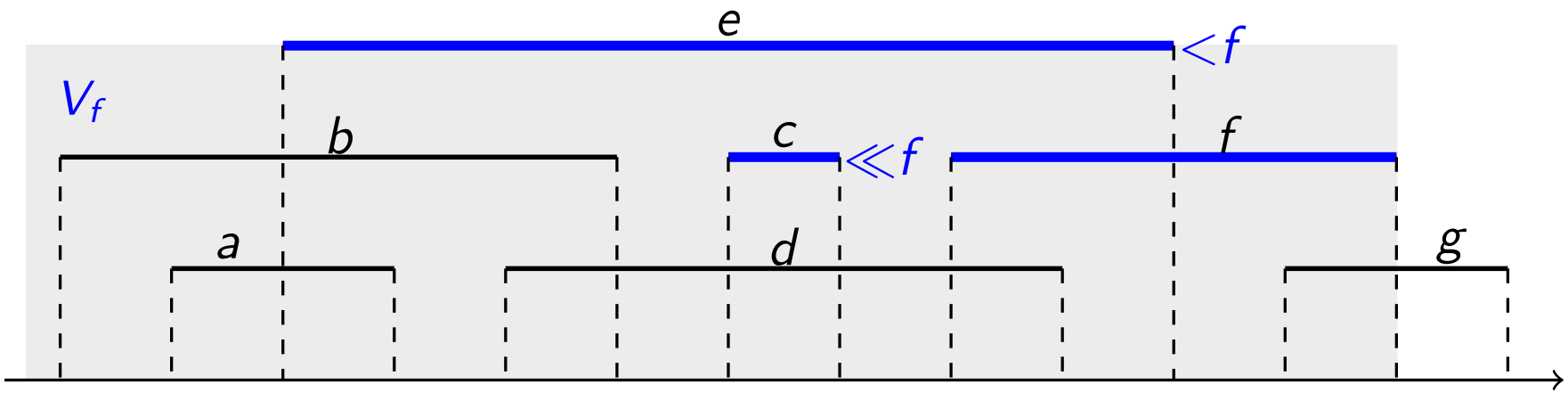

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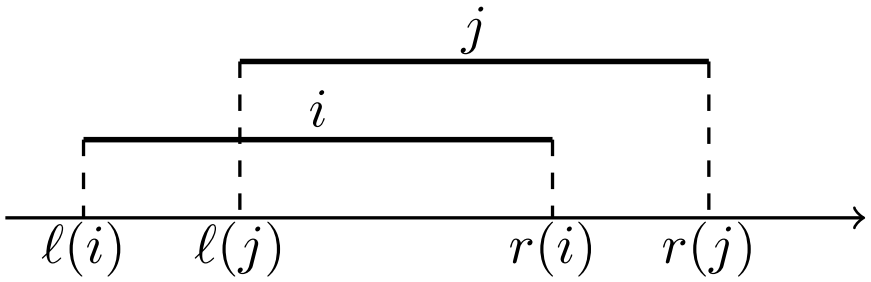

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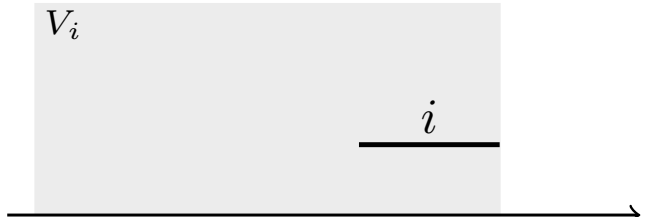
Basic sets: A, B, C

- Sets used by our dynamic programming algorithm
- A: corresponds to a maximum S-forest
- B and C: choose a solution only from a predescribed set
- $x, y \in V - V_i$ such that $x <_\ell y$

$$A_i = \max_w \{X \subseteq V_i : G[X] \in \mathcal{F}_S\}$$



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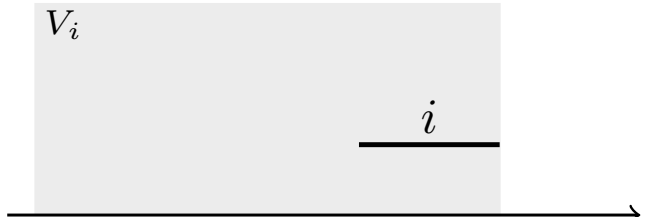

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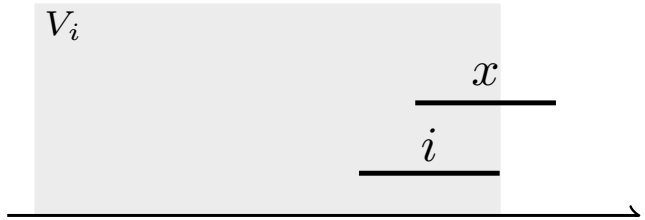

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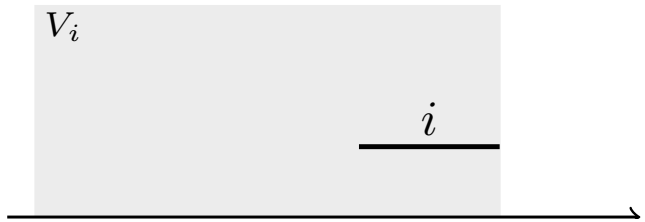

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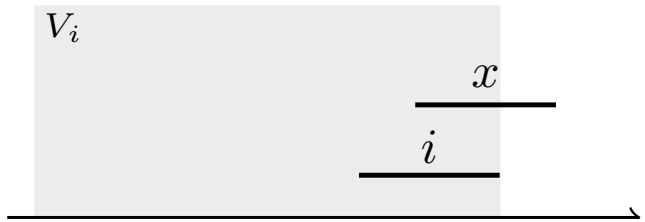

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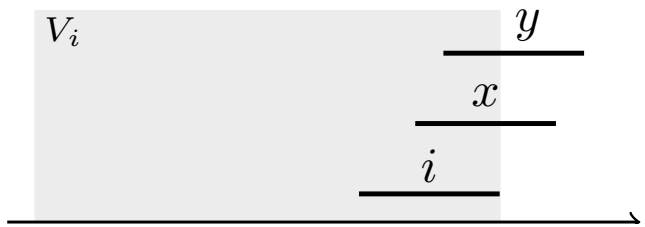
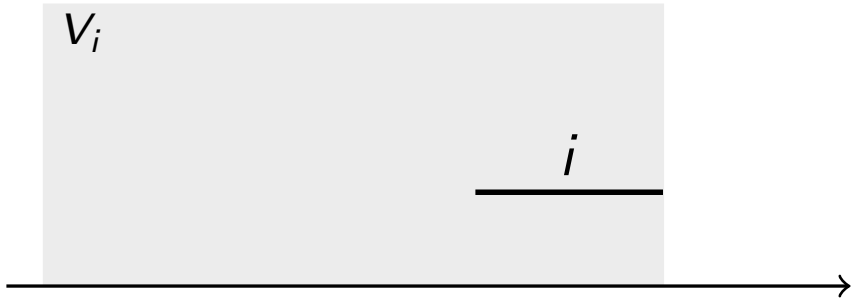


Recursive formulation for A_i

A-set:

$$A_i = \max_w \left\{ A_{<i},\ B^i_{<i} \cup \{i\} \right\}$$



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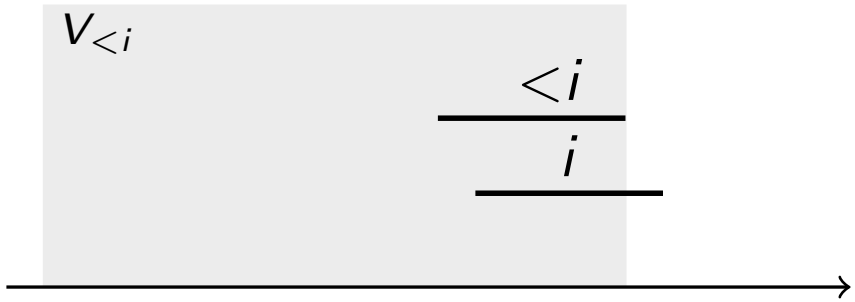

- If $i \notin A_i$ then i is irrelevant $\Rightarrow A_i = A_{<i}$
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Recursive formulation for B_i^x

Let $x' = \ell\text{-}\min\{i, x\}$ and let $y' = \{i, x\} \setminus x'$:

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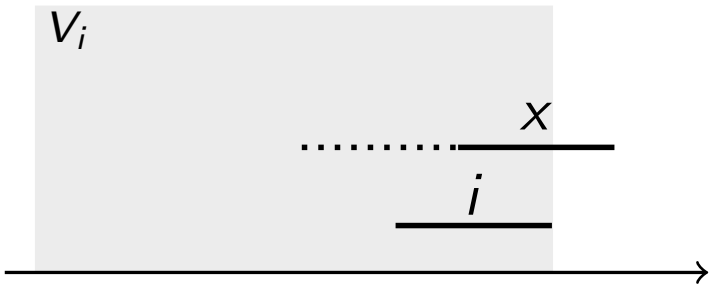

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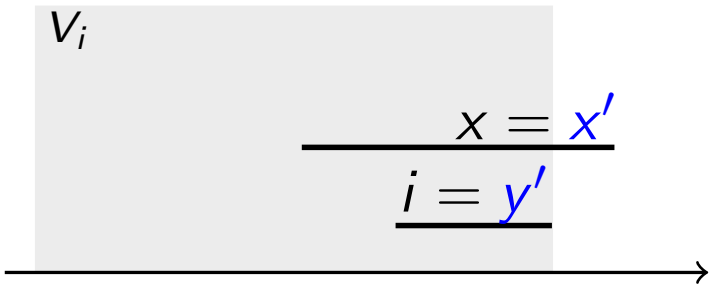

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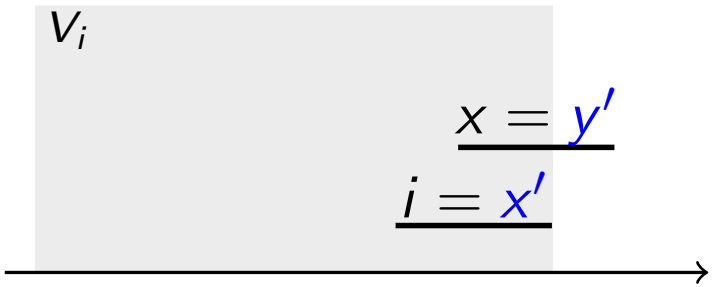

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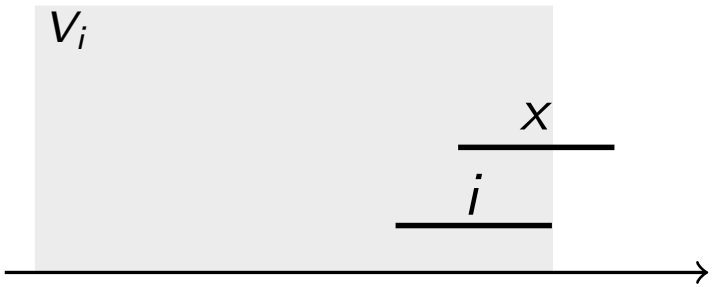

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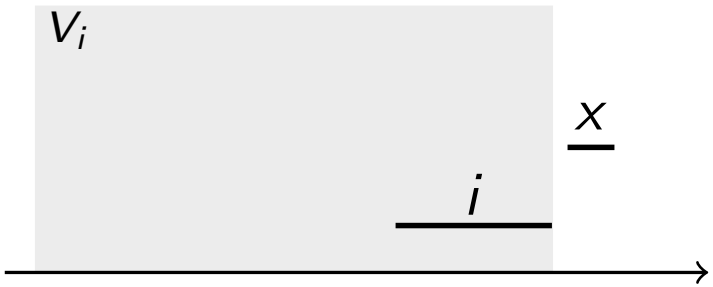

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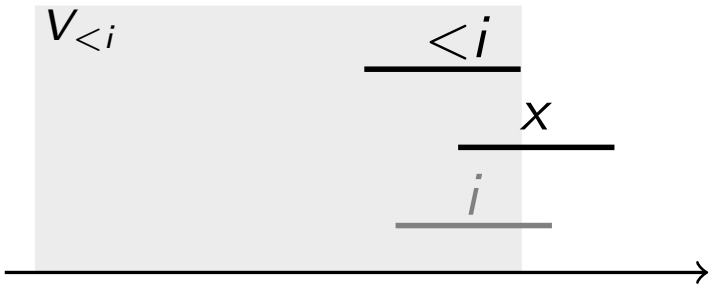

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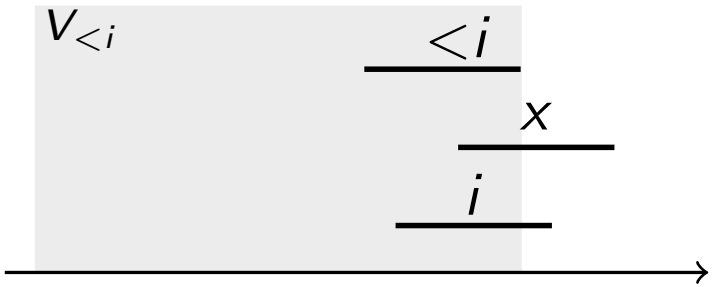

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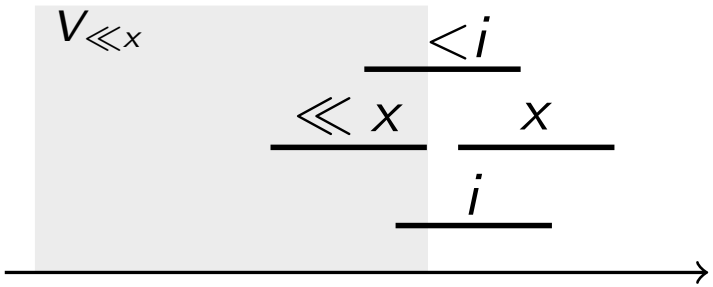

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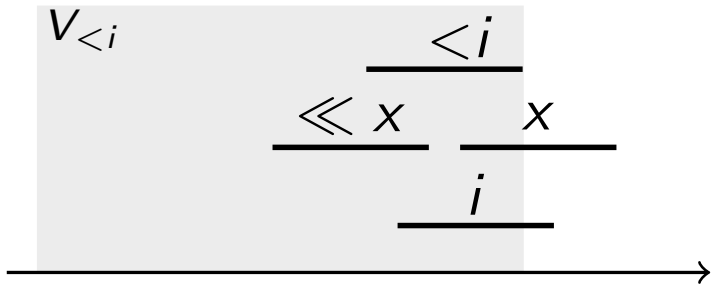

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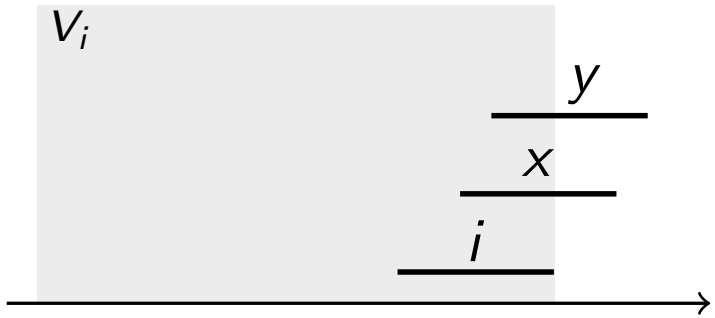

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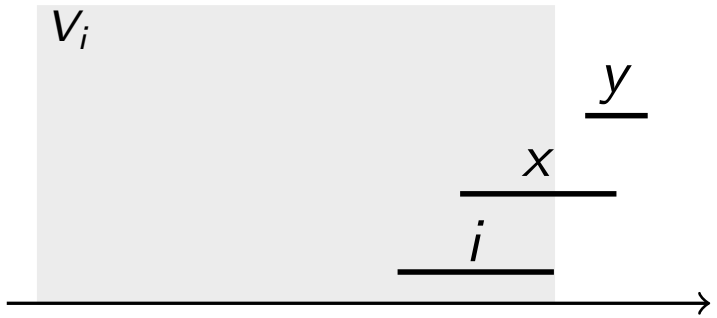

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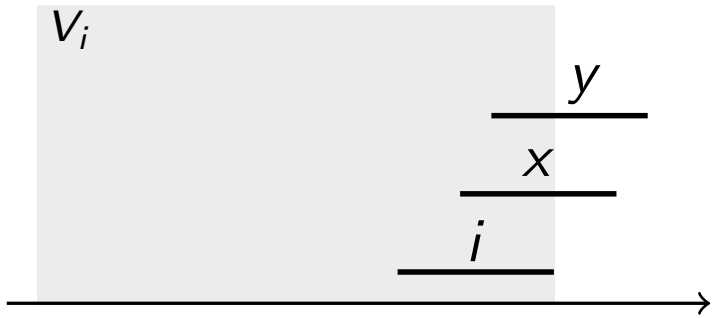

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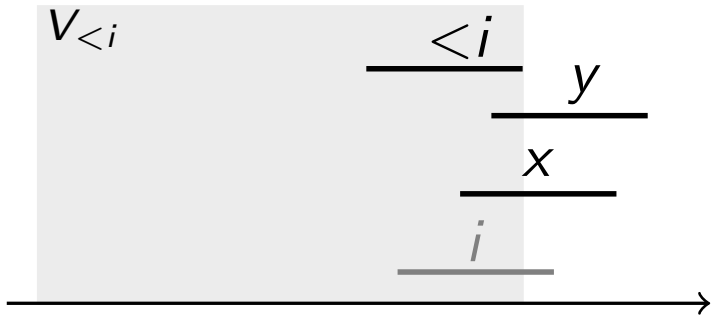

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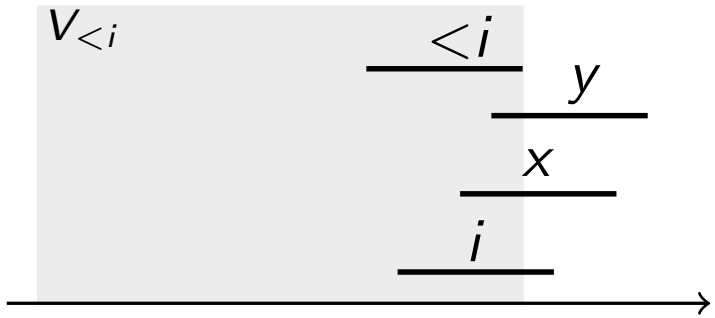

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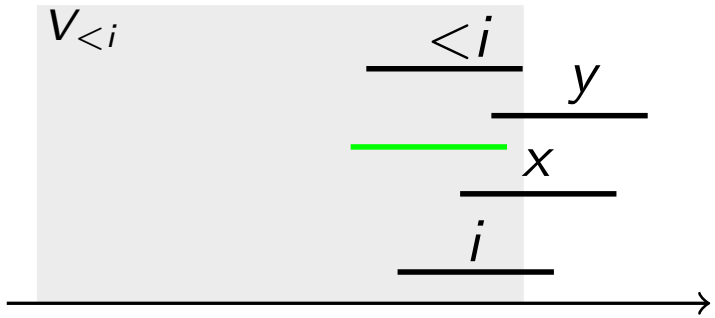

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  - $C_{<i}^{i,x} \cup \{i\}$: none of its subset can induce an S-cycle with y

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There is a poly-time algorithm that computes SFVS of an interval graph.

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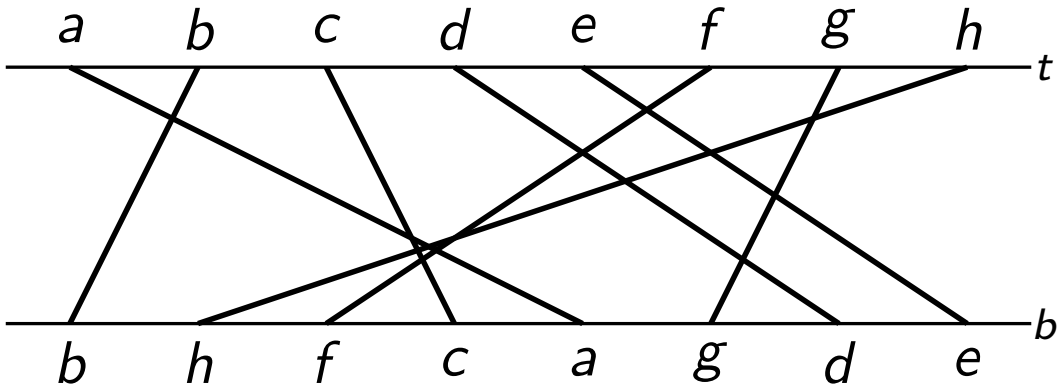
The total running time of the algorithm is $O(n^3)$.

Permutation Graphs



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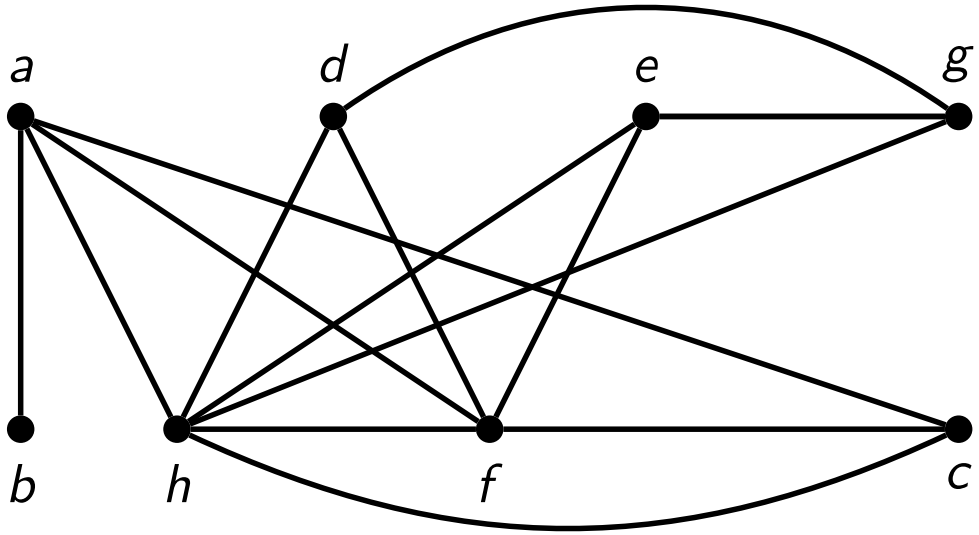

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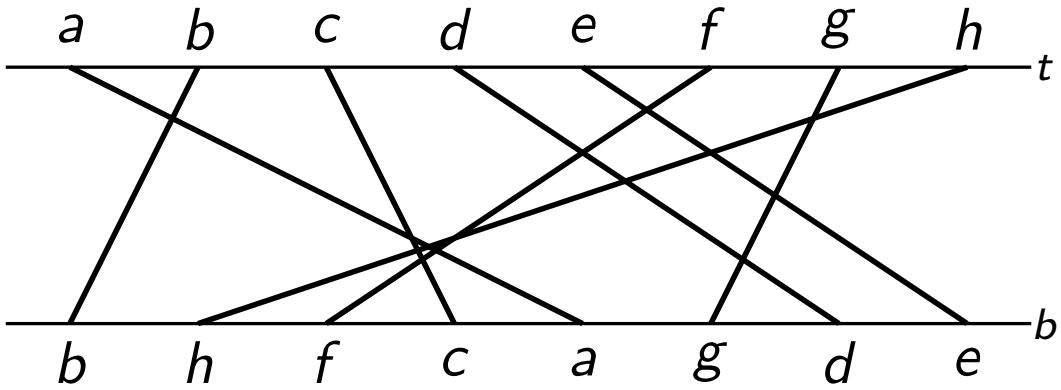

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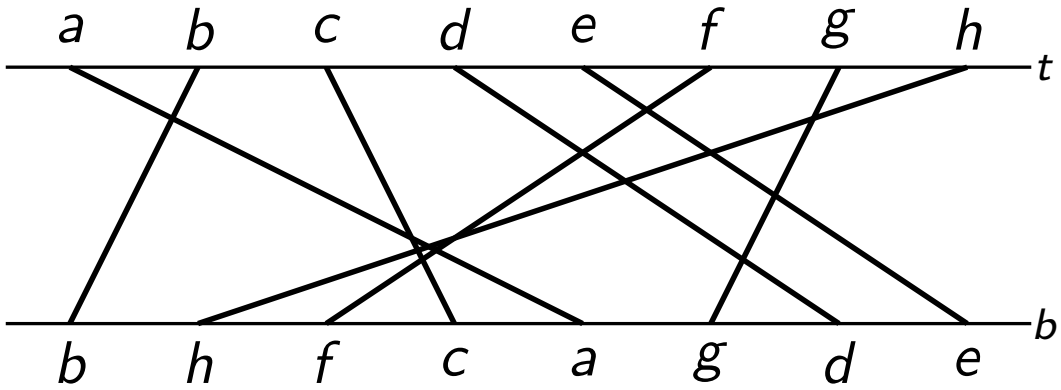

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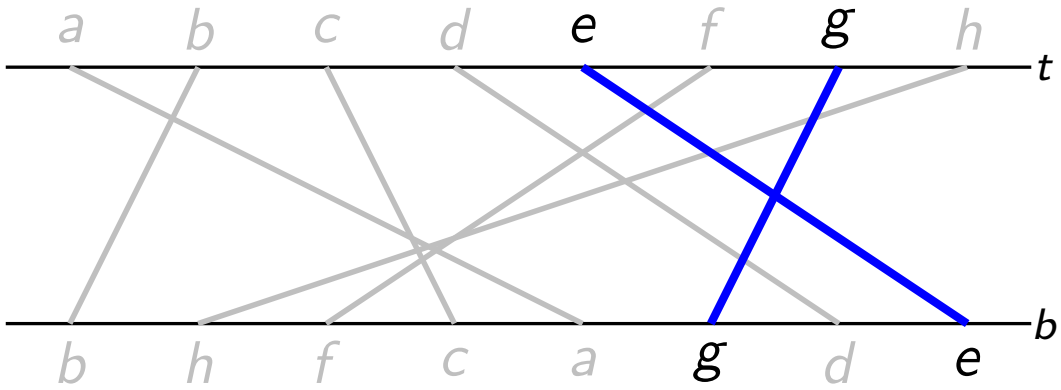
Crossing Pairs



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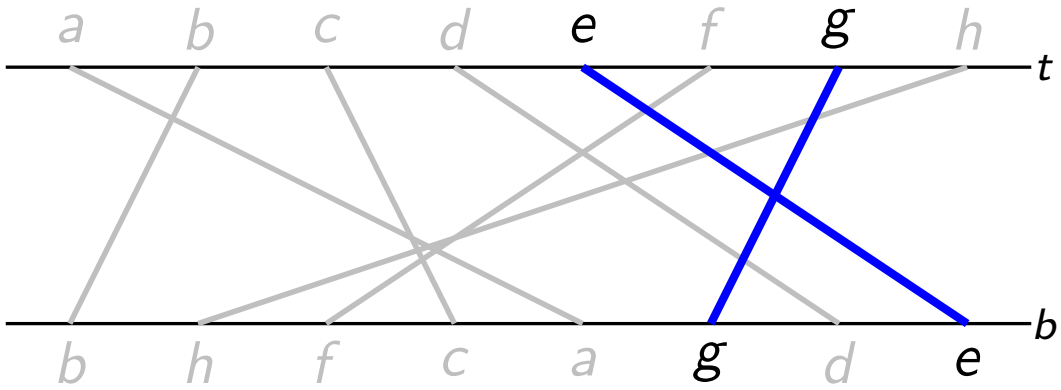

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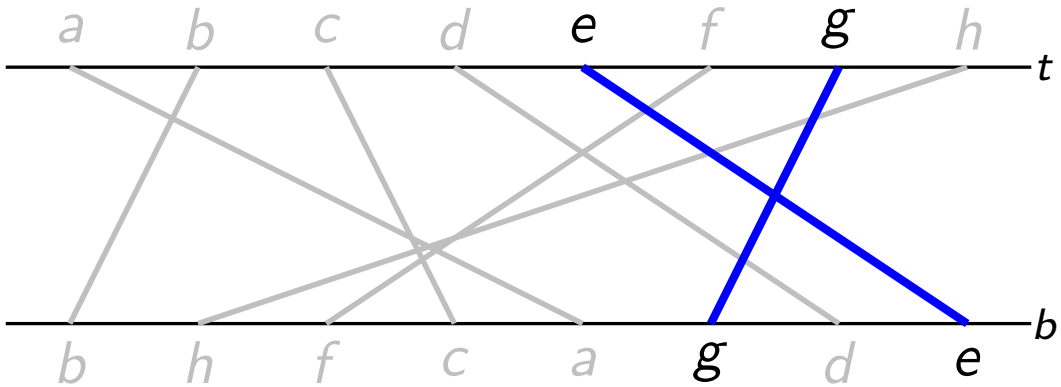

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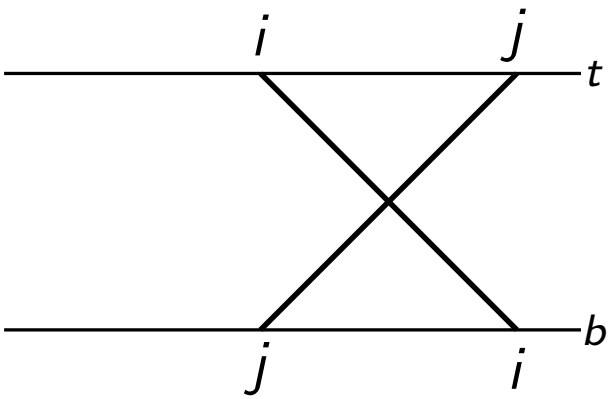

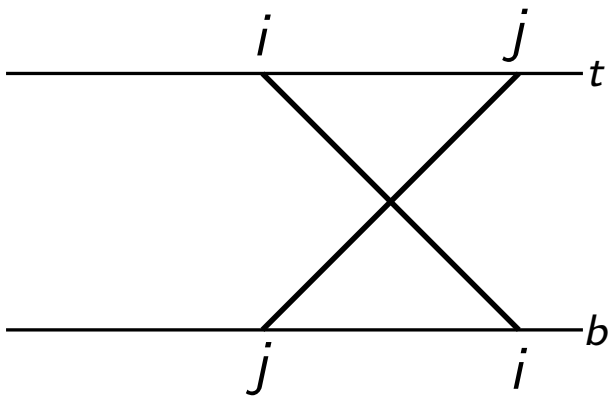

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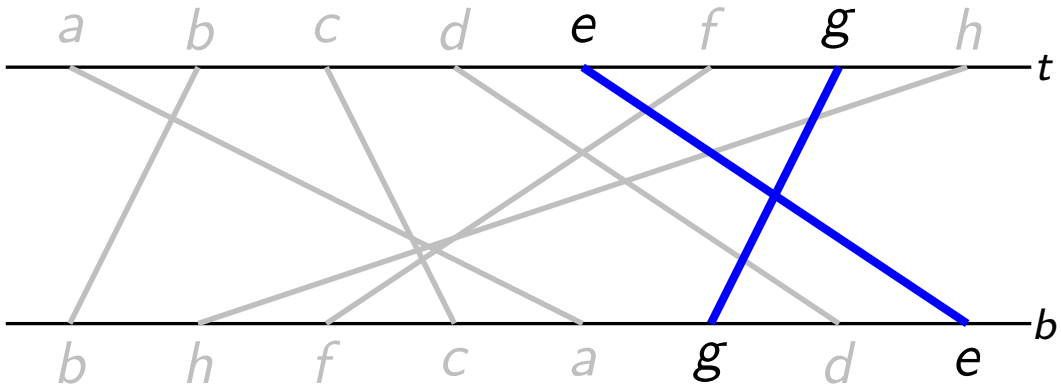

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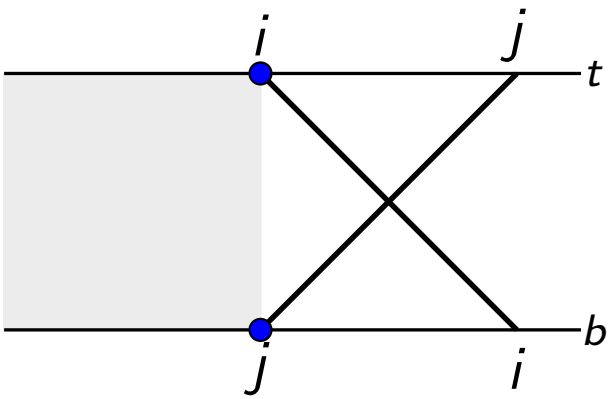

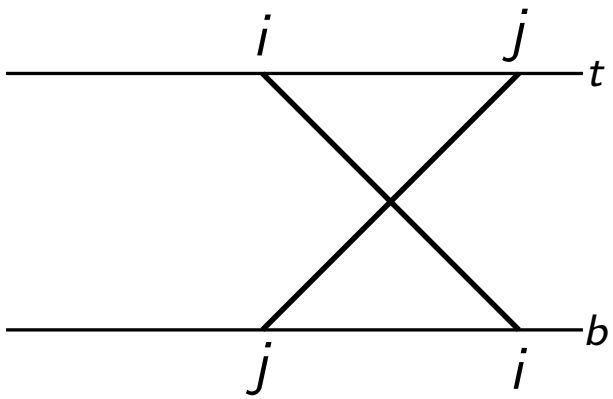

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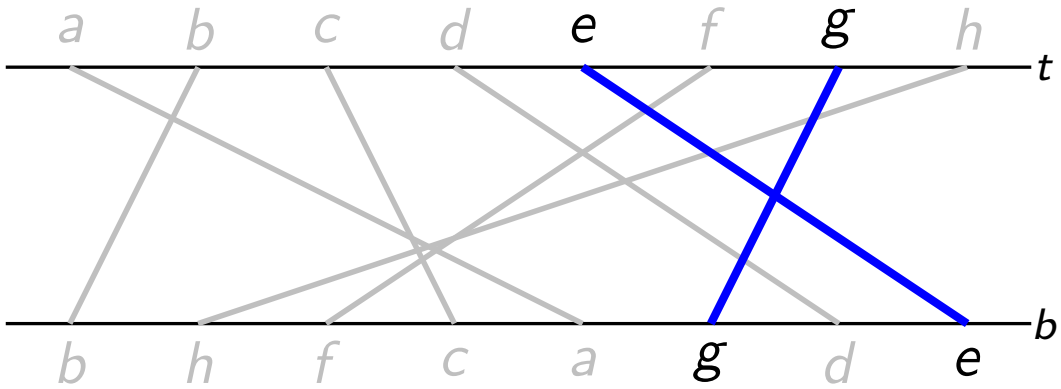

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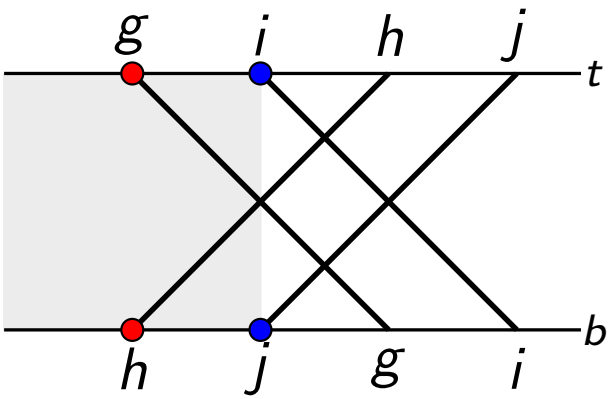

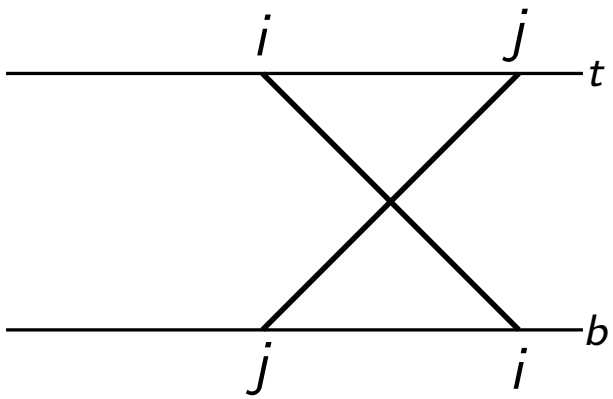

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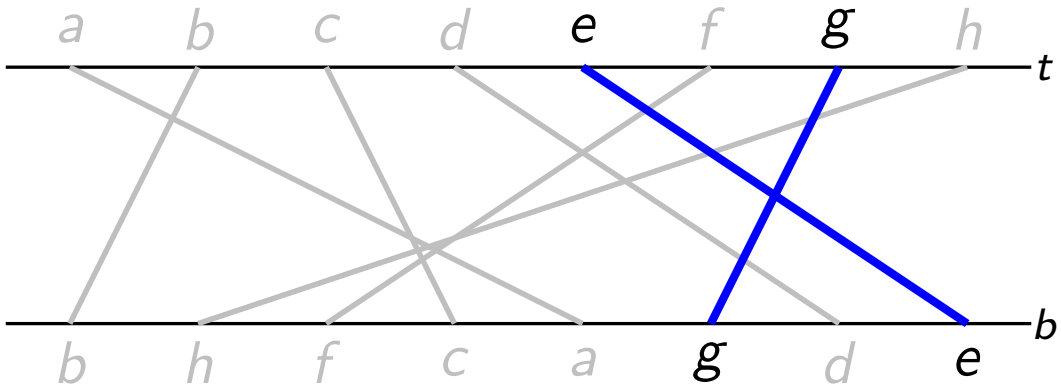

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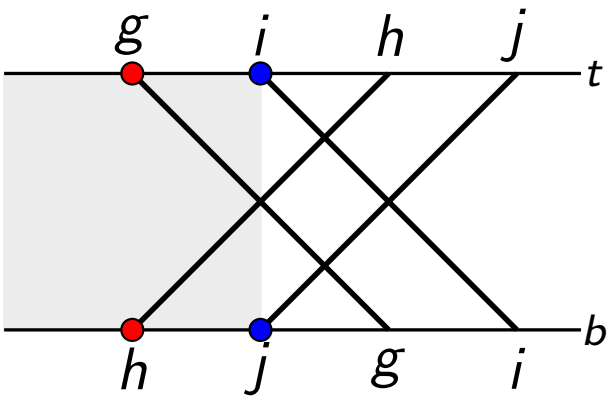

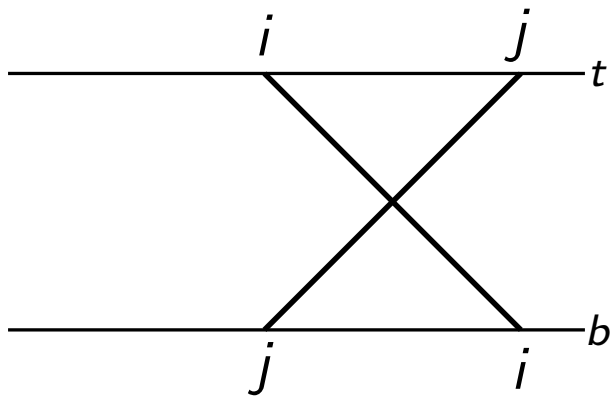

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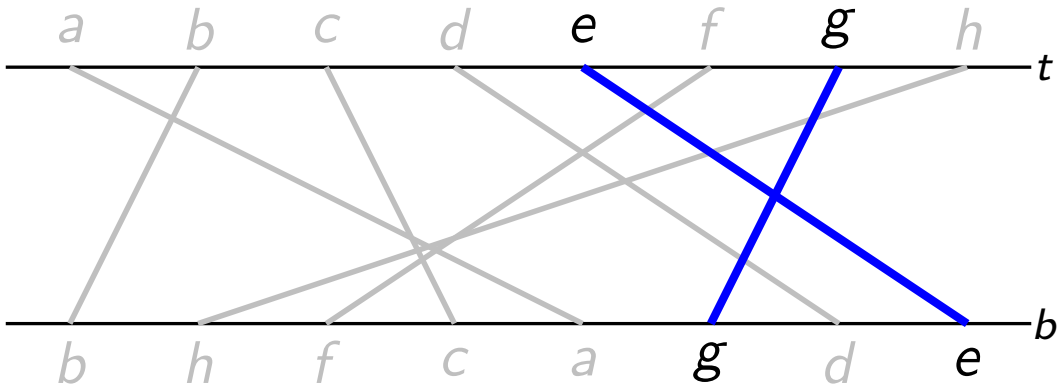

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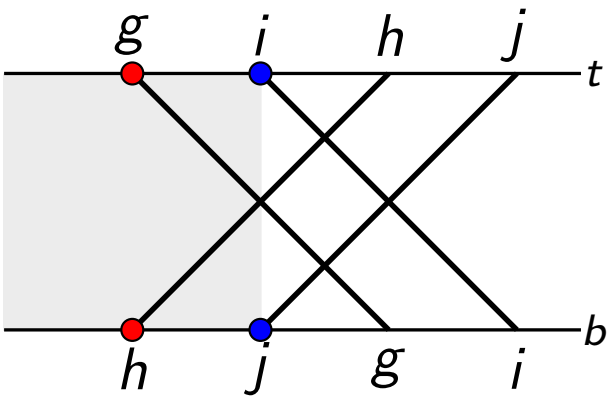

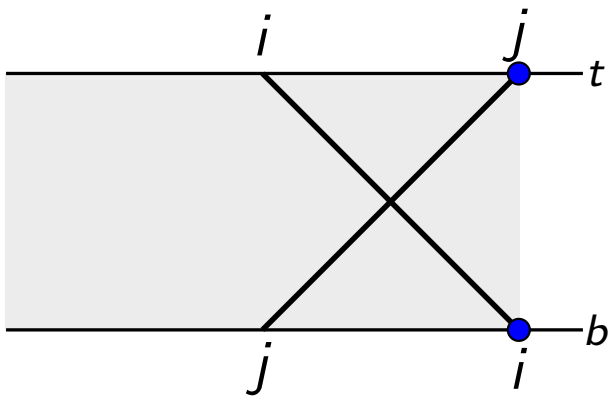

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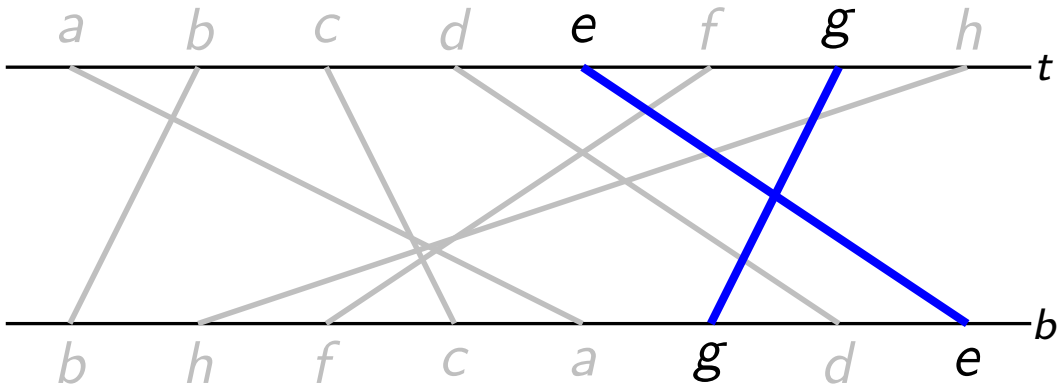

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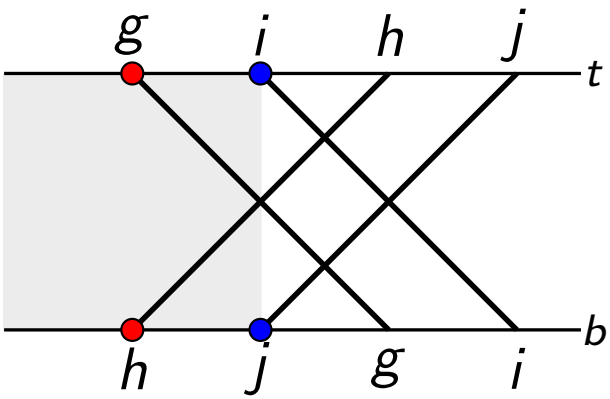
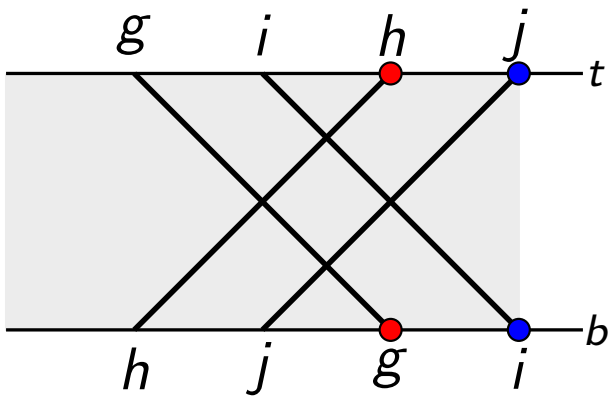
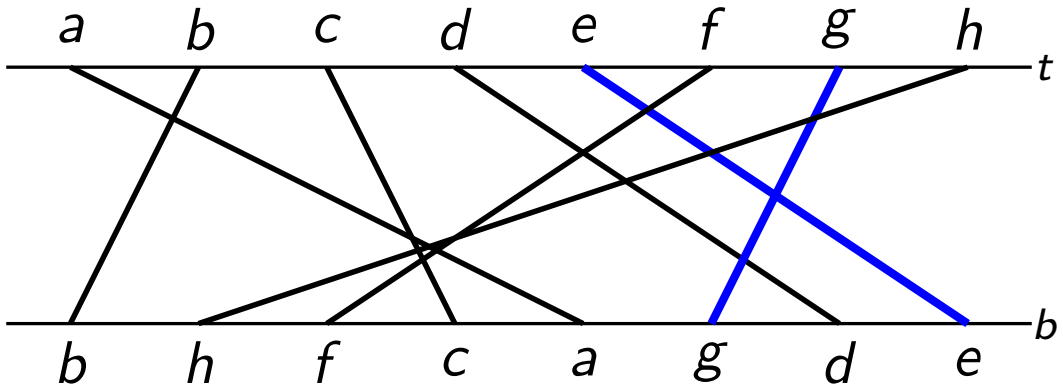




Predecessors of Crossing Pairs



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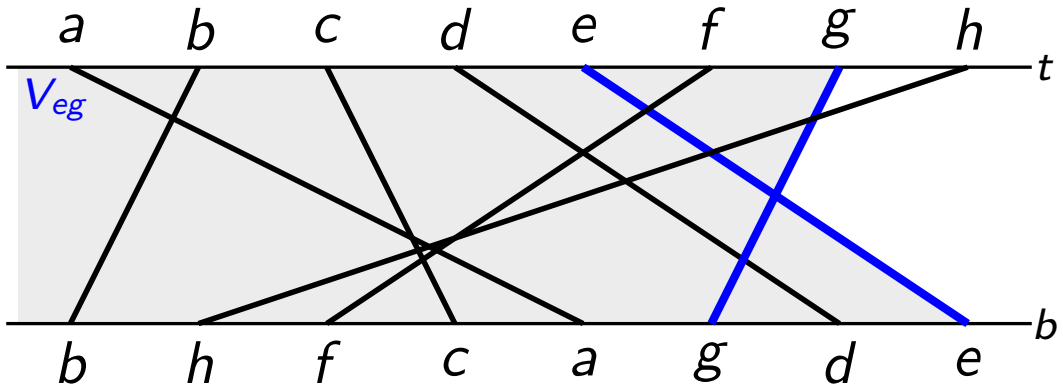

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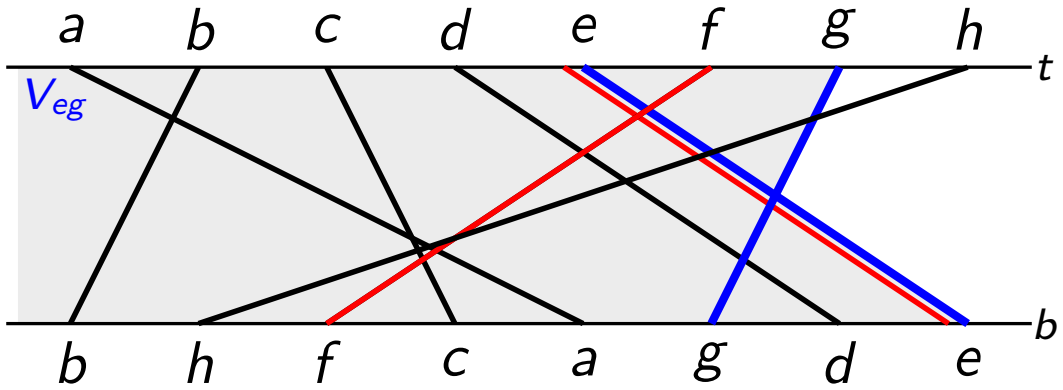

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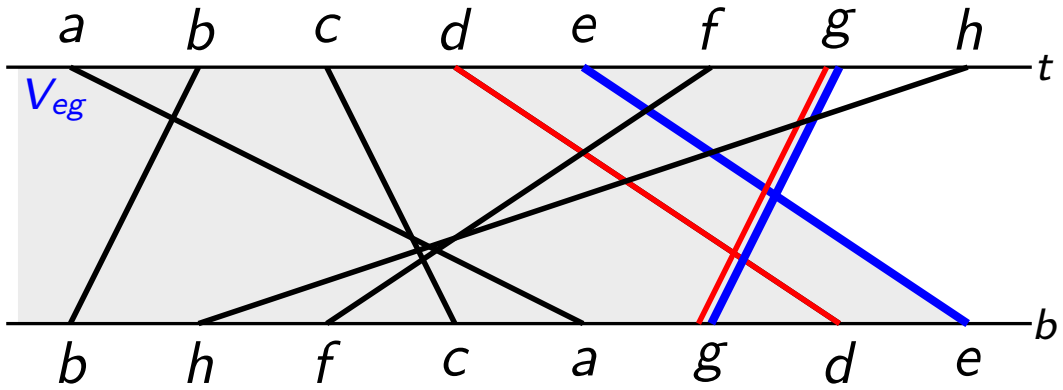

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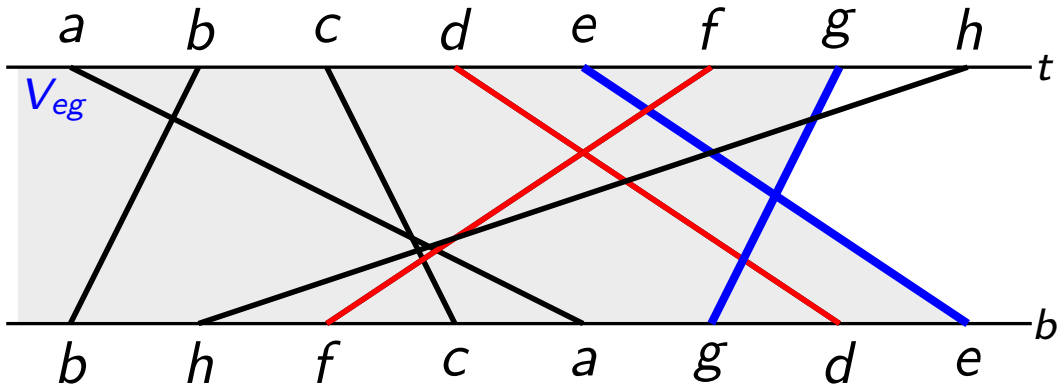

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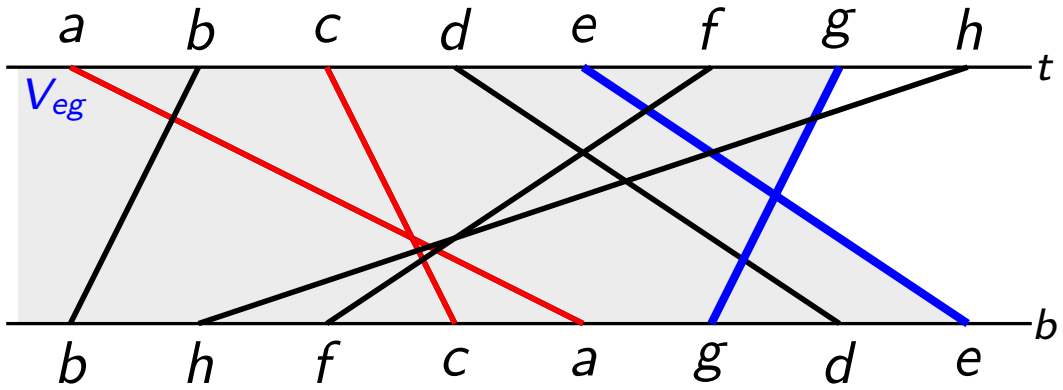
Predecessors of Crossing Pairs



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  non-adjacent xy with x and y the rightmost bottom and top

Basic sets: A, B, C

- Sets used by our dynamic programming algorithm
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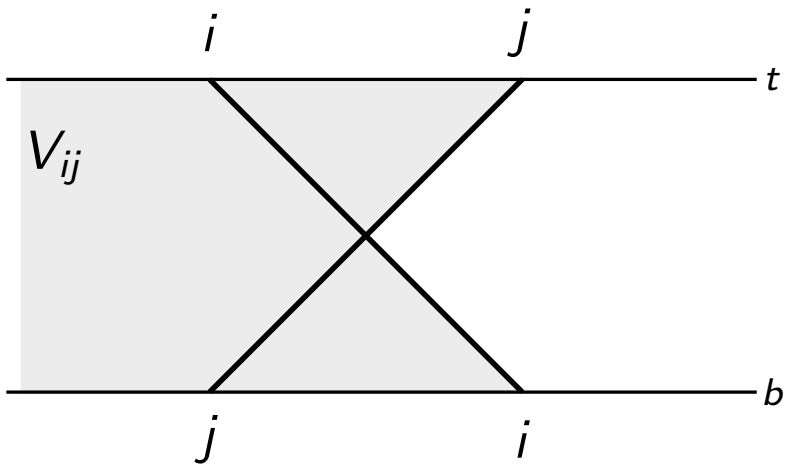

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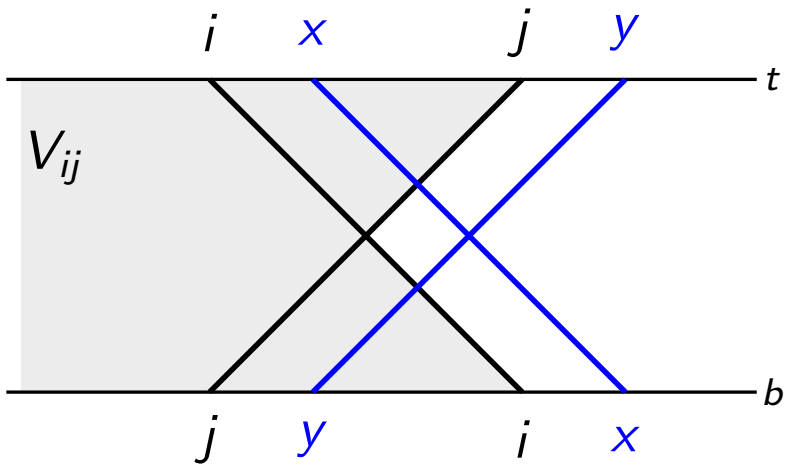

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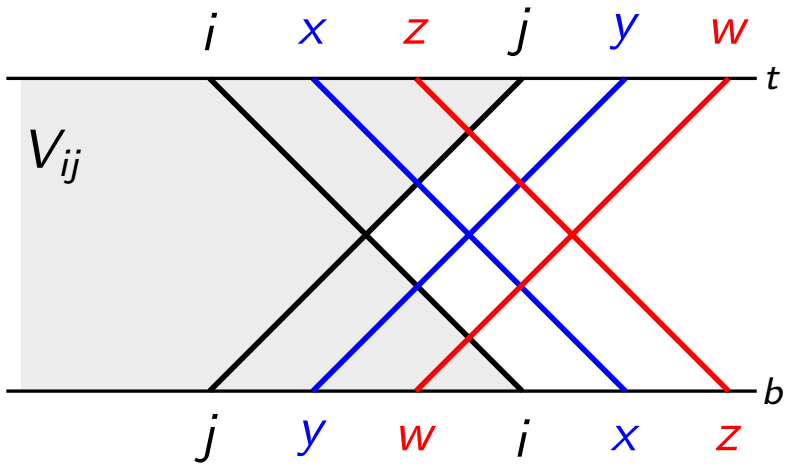

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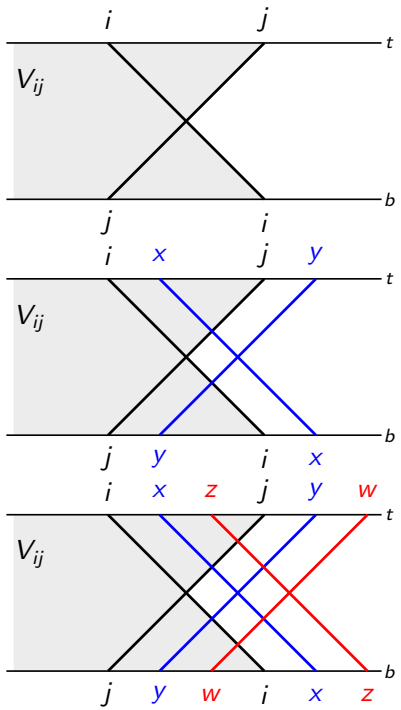

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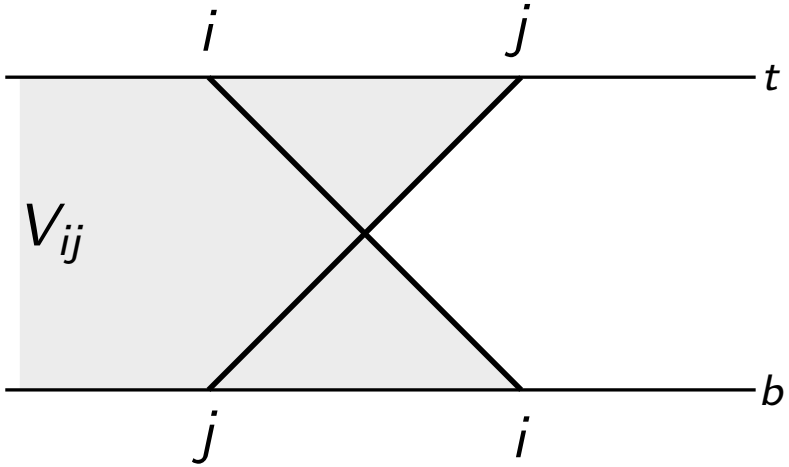

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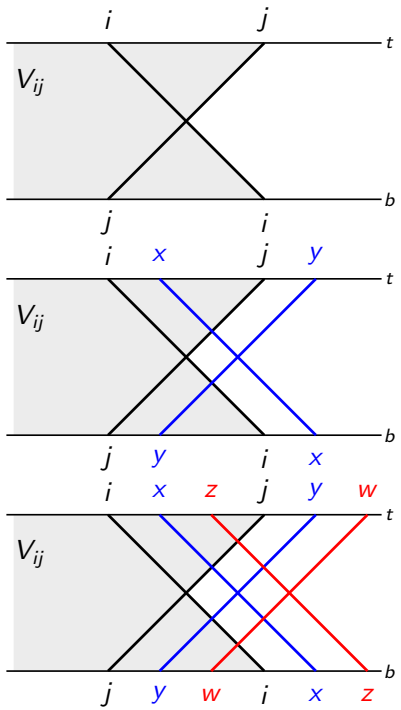

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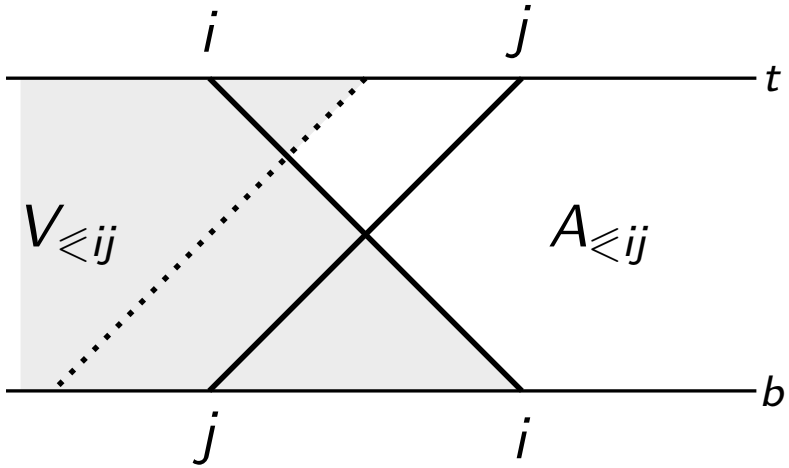

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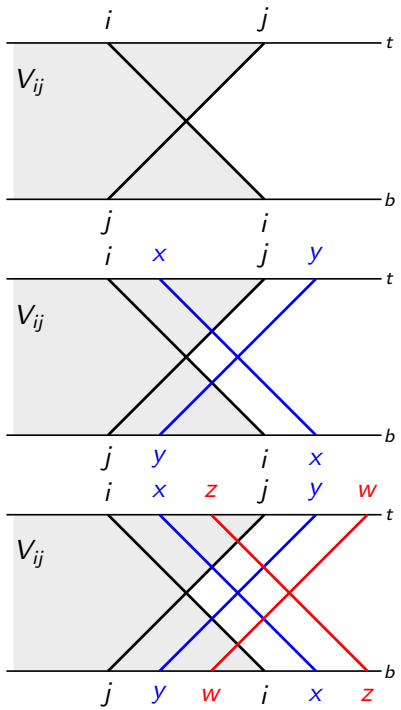

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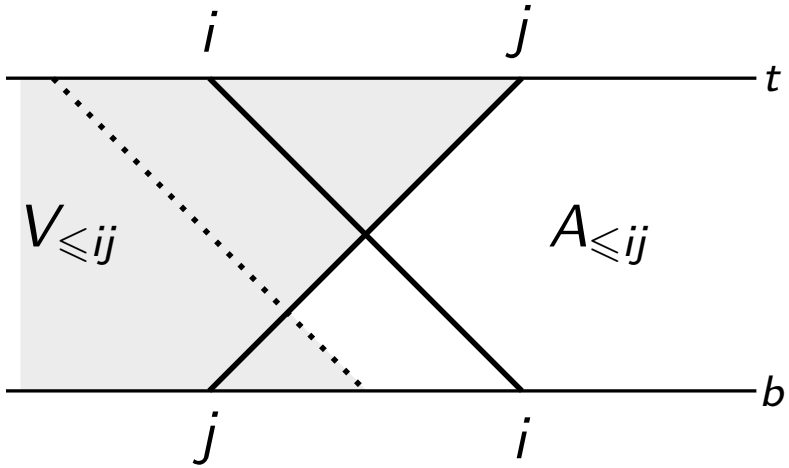

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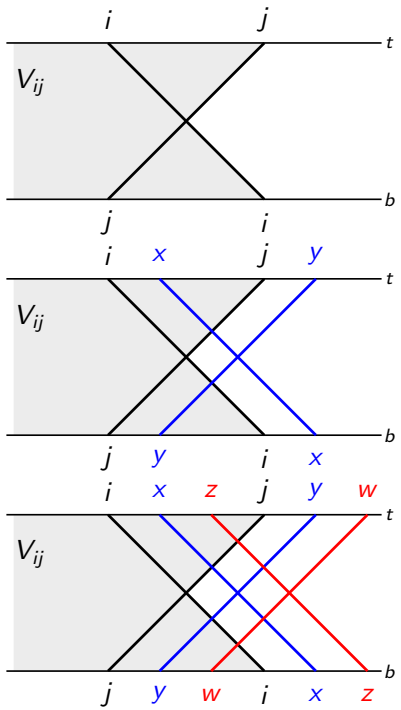

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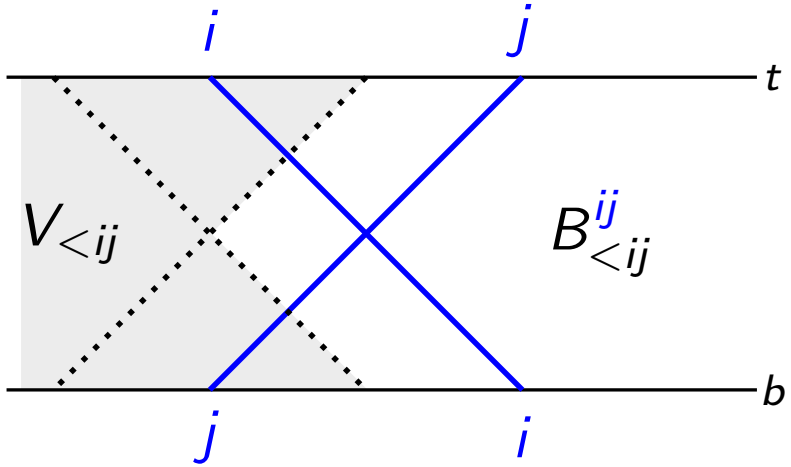

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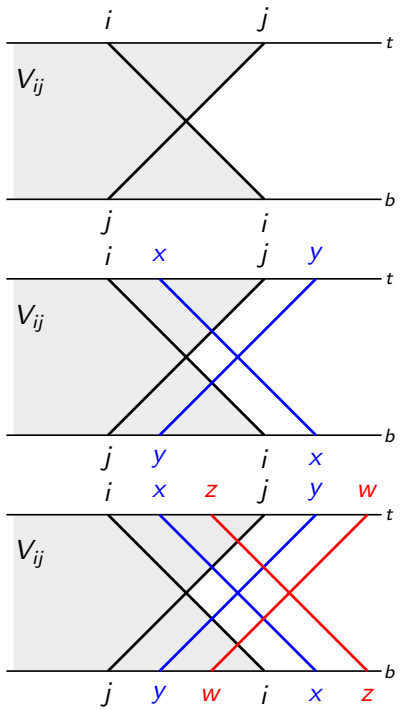

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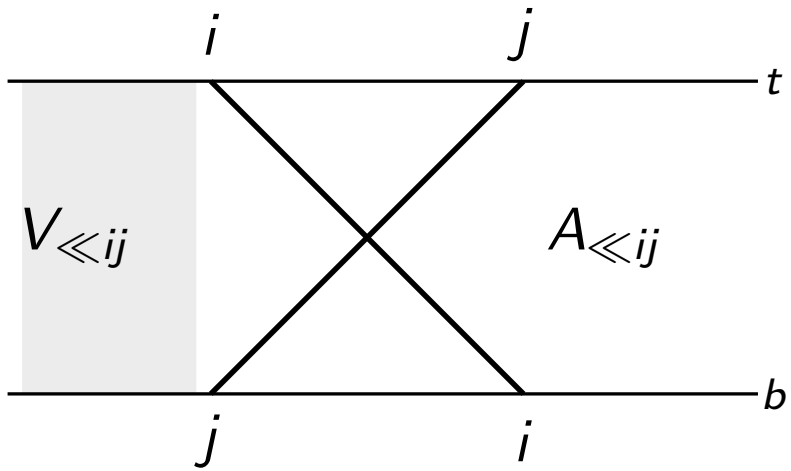

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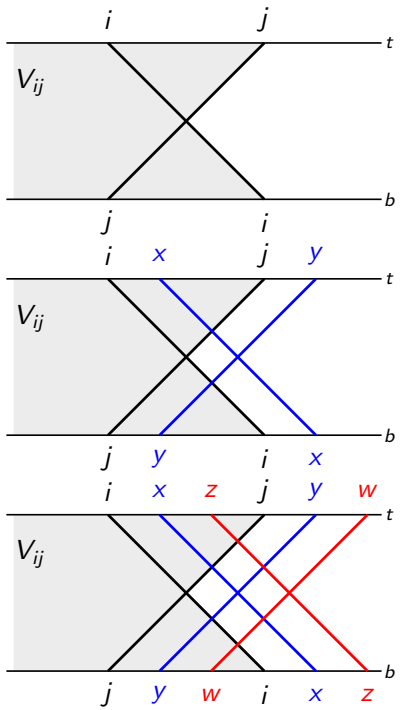

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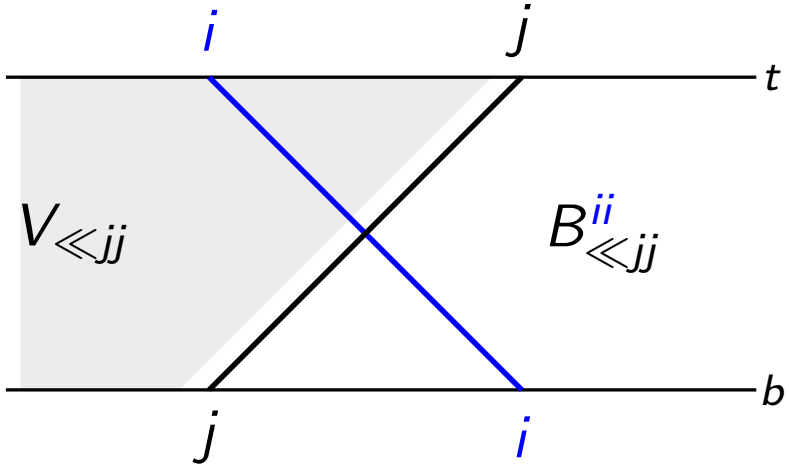

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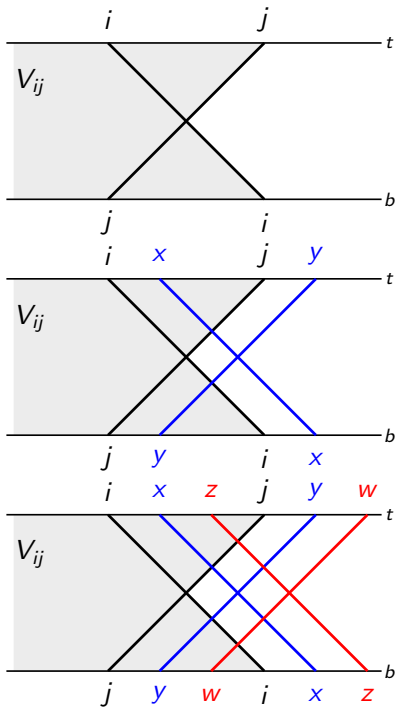

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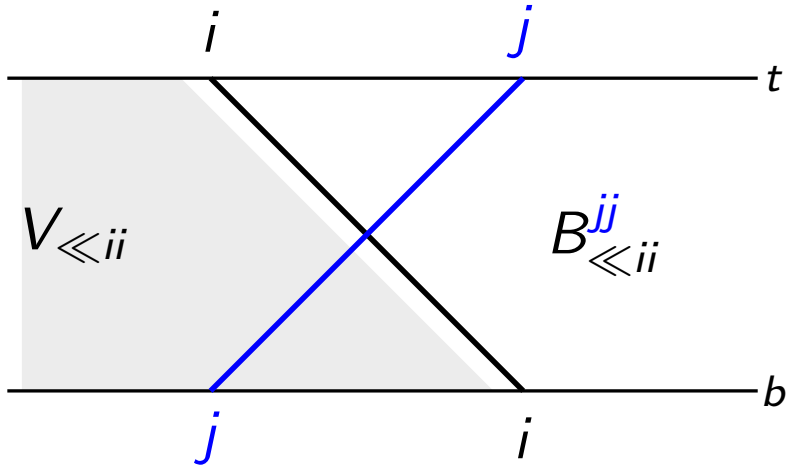

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The total running time of the algorithm is $O(n + m^3)$.

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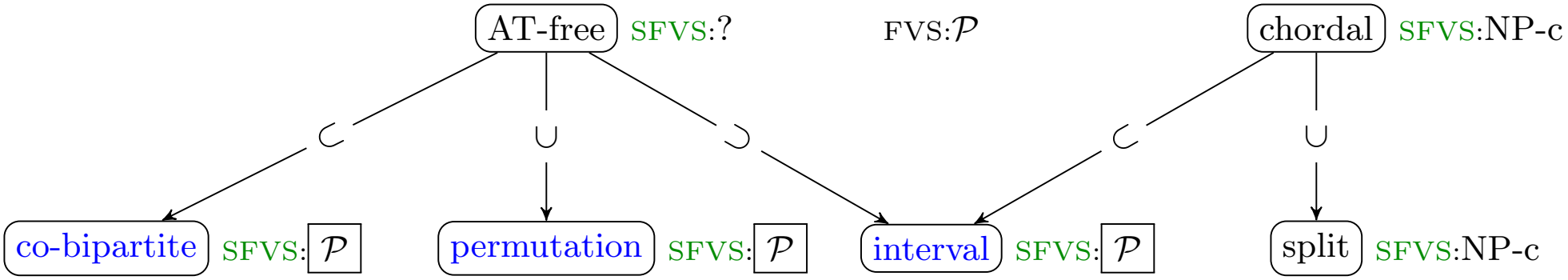

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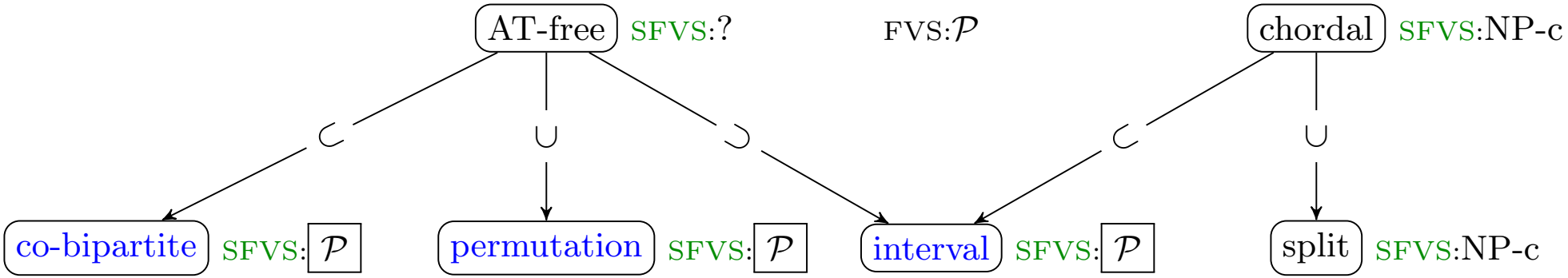

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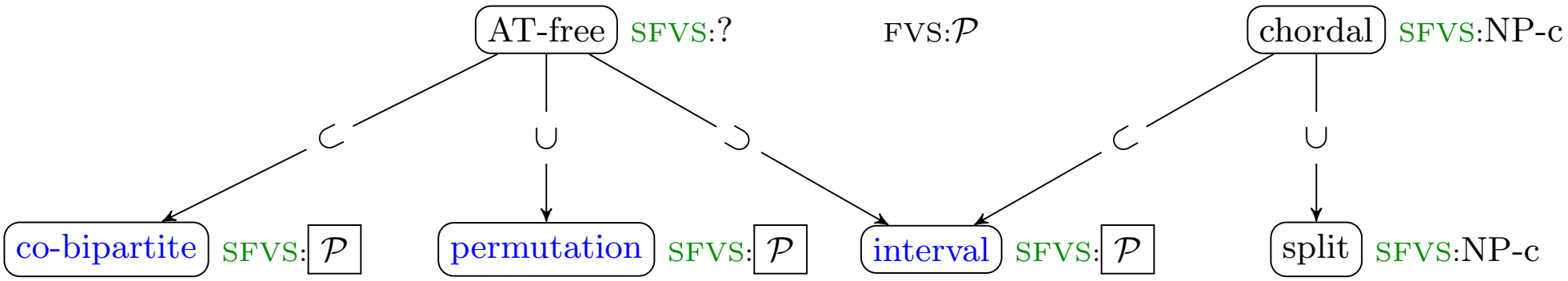

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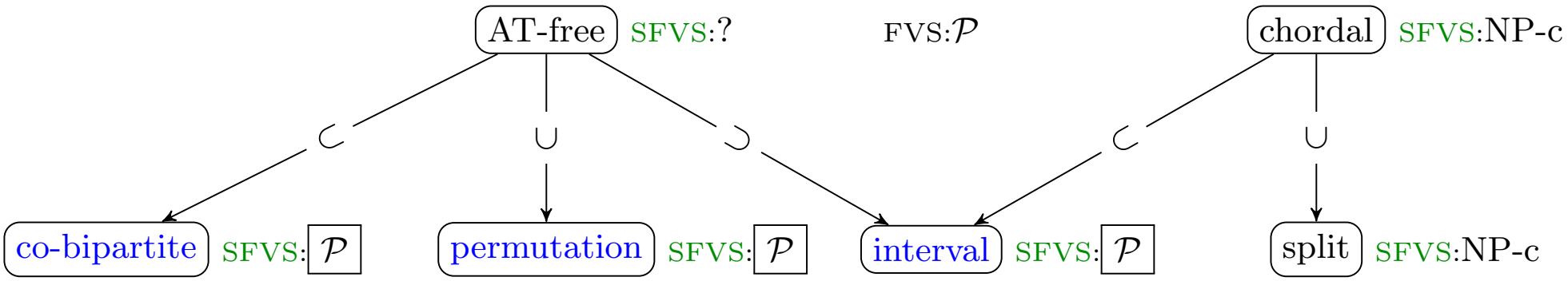

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