Reliable communication via semilattice properties of partial knowledge

A. Pagourtzis [1] G. Panagiotakos [2] D. Sakavalas [1]

[1] School of Electrical and Computer Engineering
National Technical University of Athens

[2] School of Informatics
University of Edinburgh

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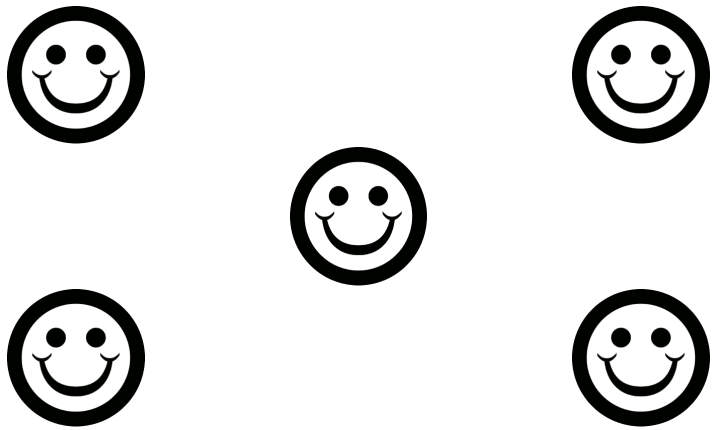
Distributed Computing in an unreliable environment



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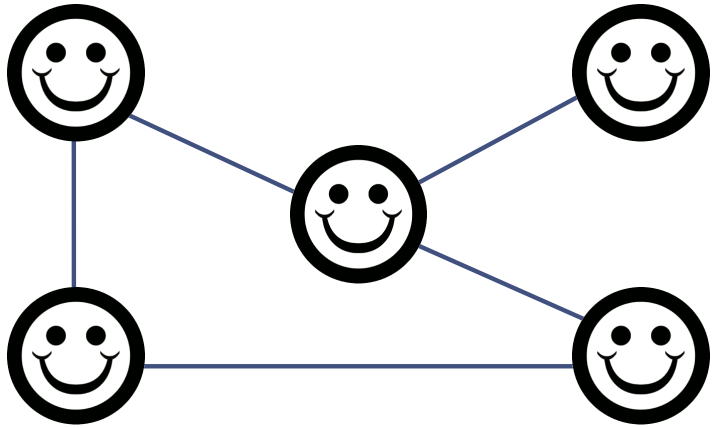

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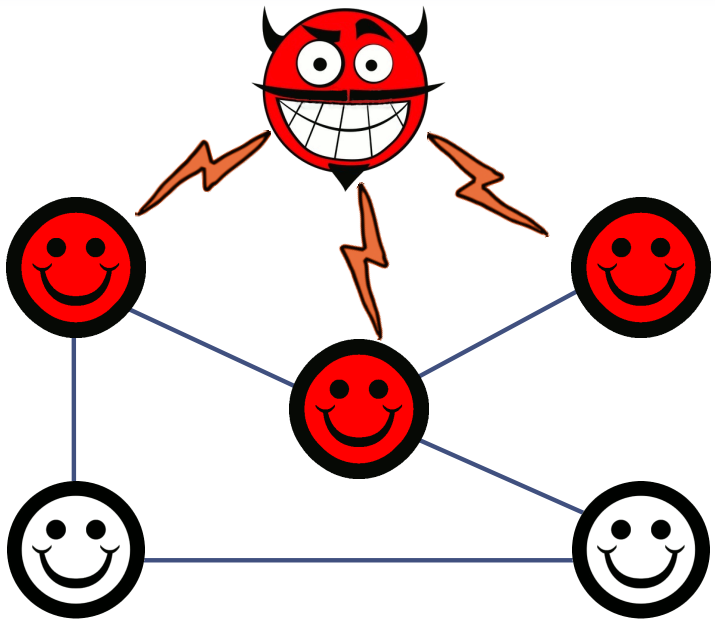

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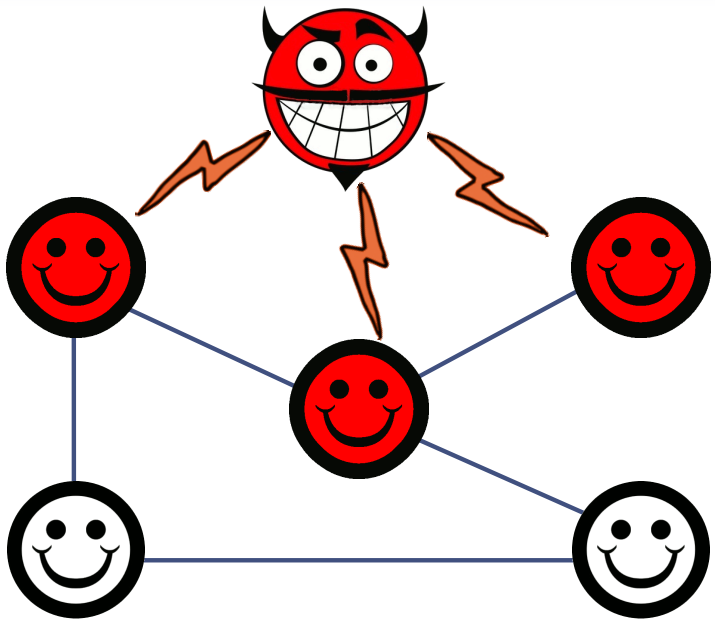

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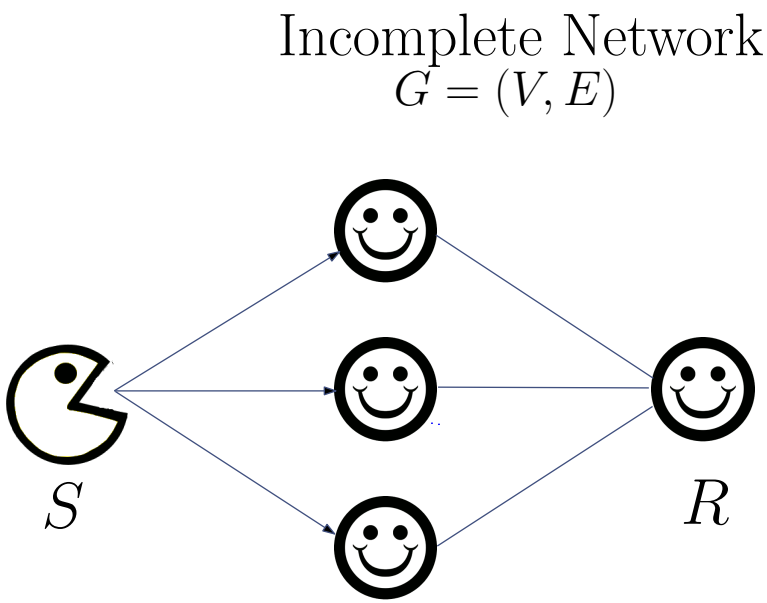
- Players arranged in a communication network G.
- Adversarial Behavior: Corrupted players controlled by a central active (Byzantine) adversary.
- Achieve goal despite the presence of corruptions.

Reliable Communication

– Reliable Message Transmission (RMT) problem:
Correct delivery of message x from **Sender** S to **Receiver** R, despite the existence of corrupted players.



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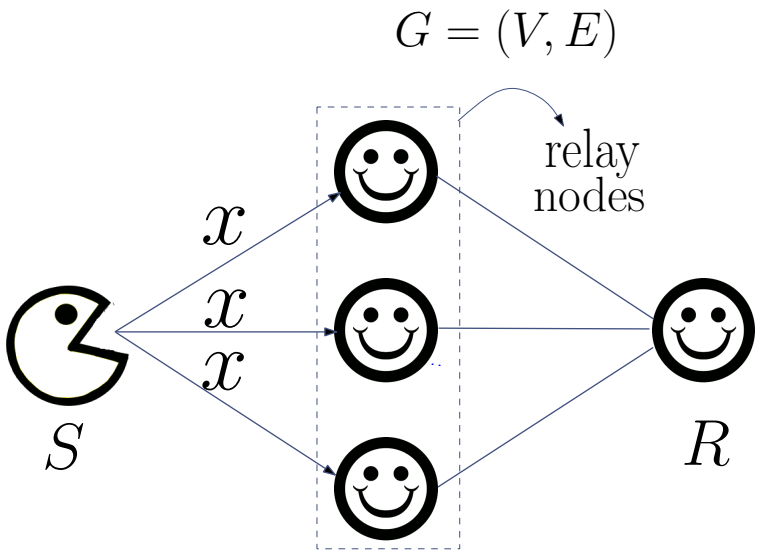

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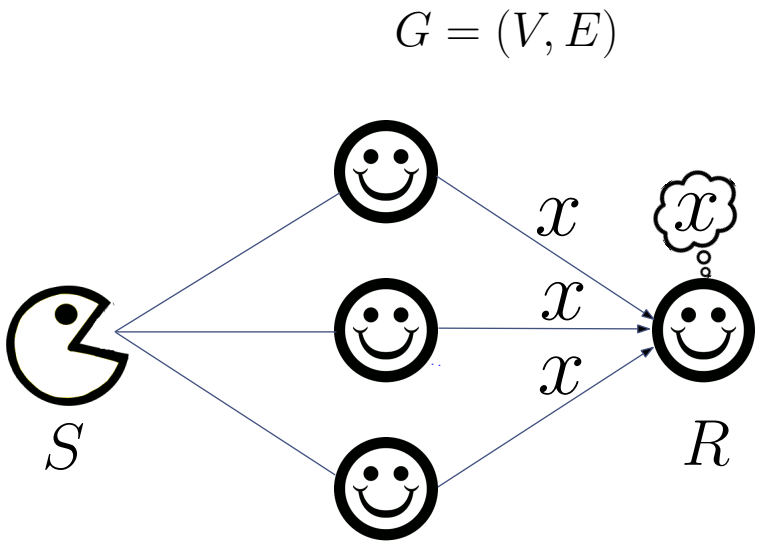
(**Sender's input**: x, **Receiver's output** (decision): x)



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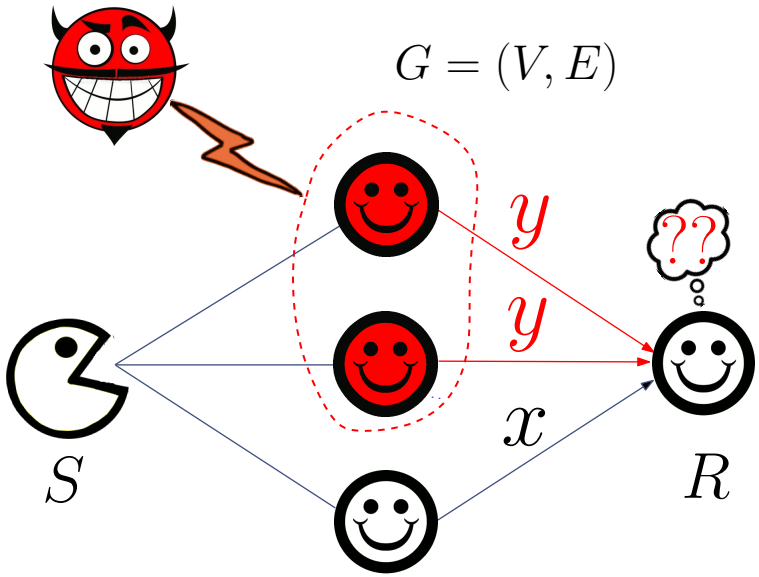

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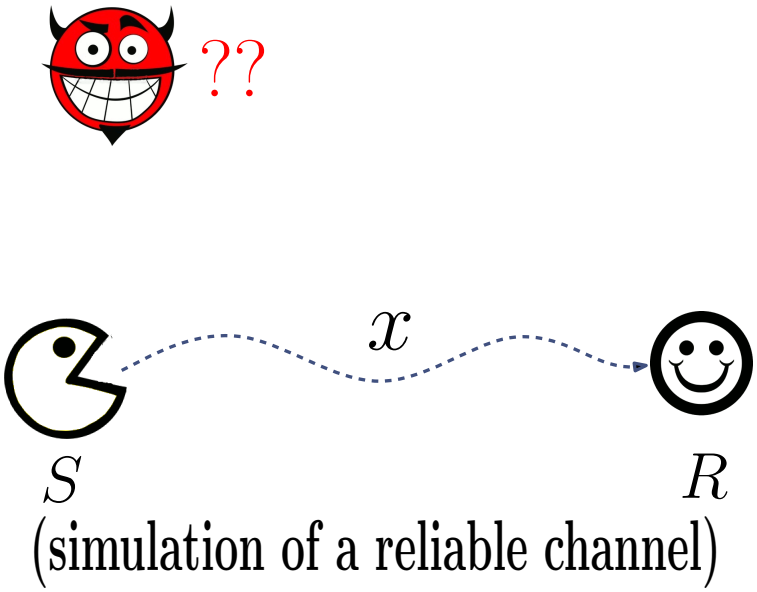

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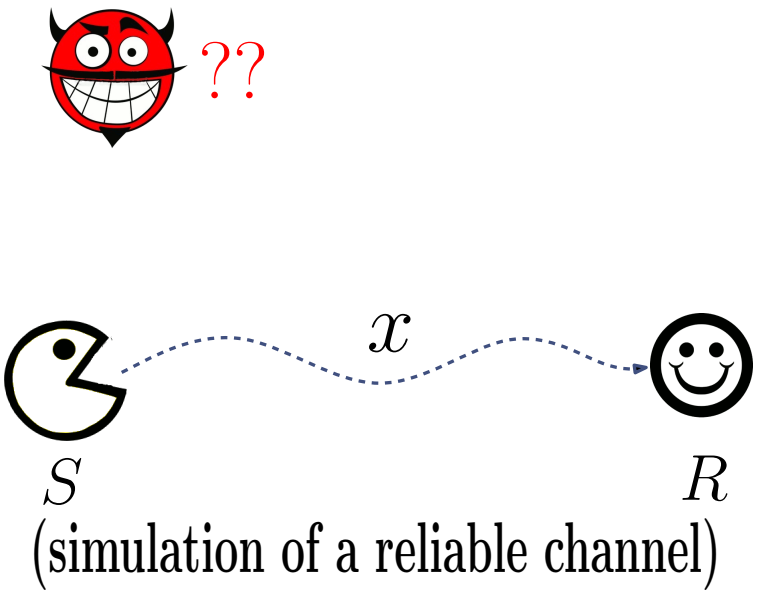
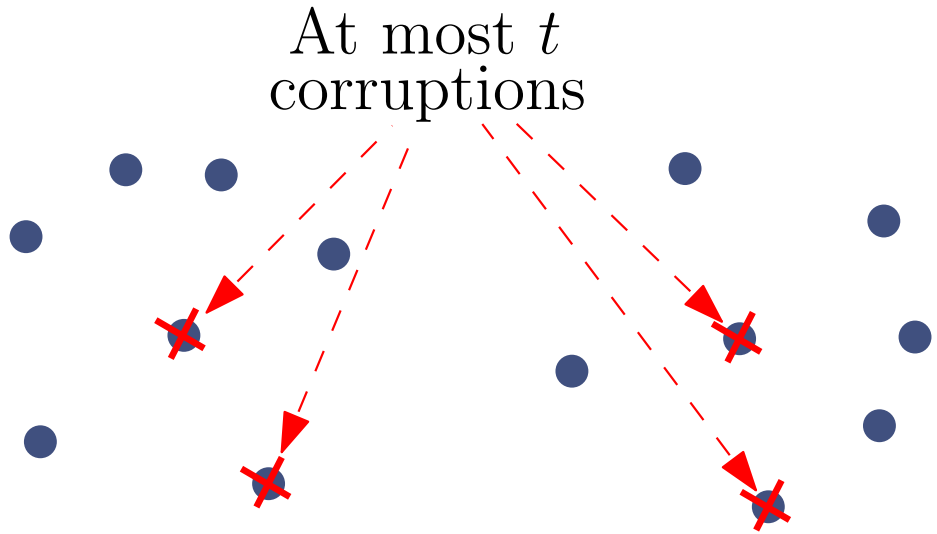


– Main result: Exact characterization of instances where RMT is feasible (impossibility condition, matching algorithm)

The Adversary

Corruption sets

– t-Global [Lamport, Shostak, Pease, '82]:
At most t corruptions.



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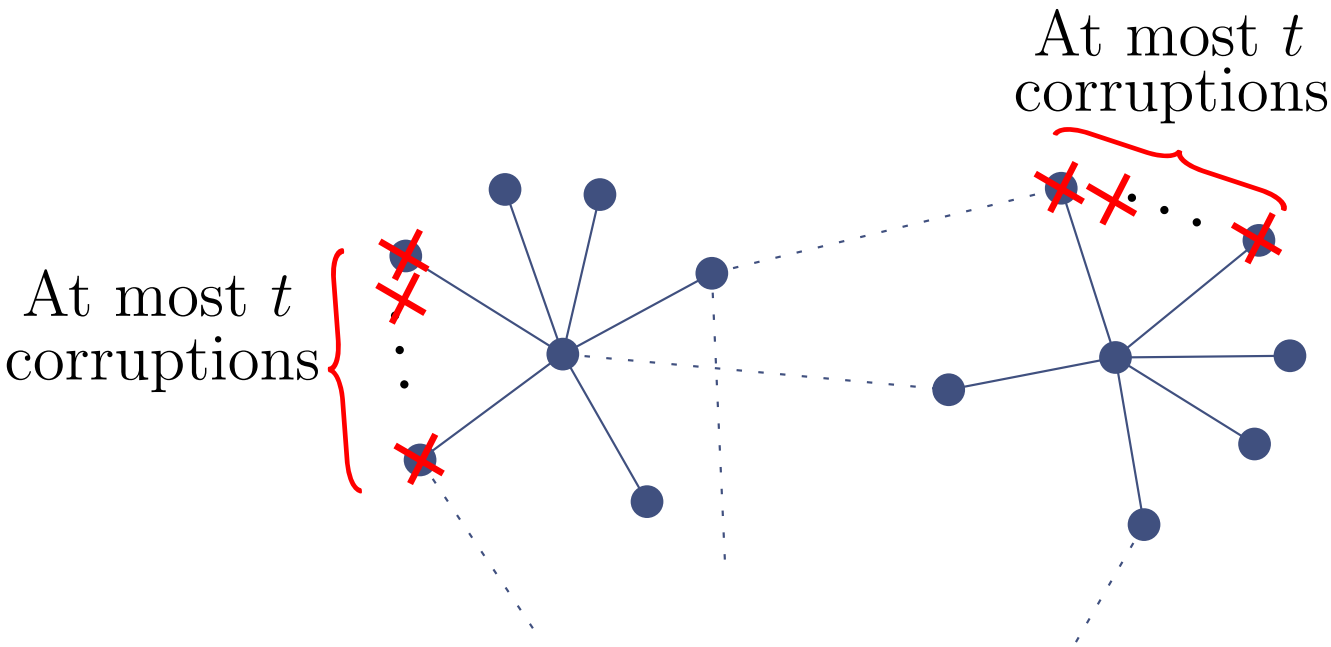

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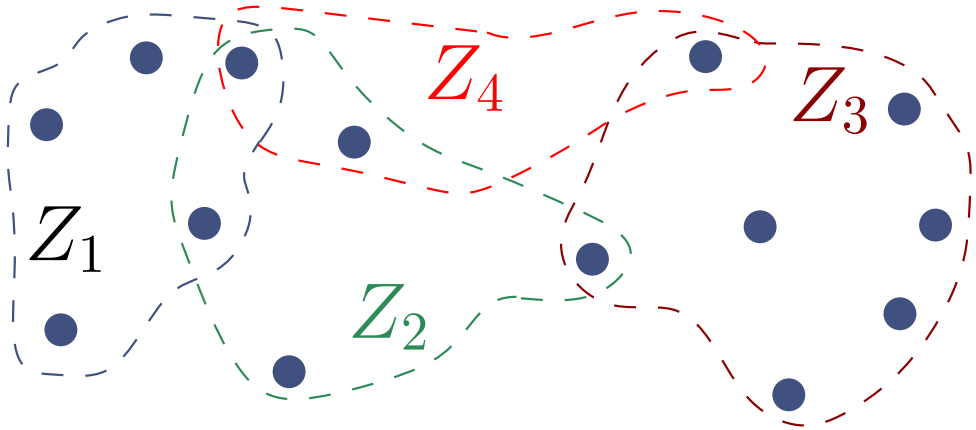
- General Adversary [Hirt, Maurer, '97]:
  Defined by the monotone family of all possible corruption sets $\mathcal{Z} \subseteq 2^V$ (**adversary structure**).

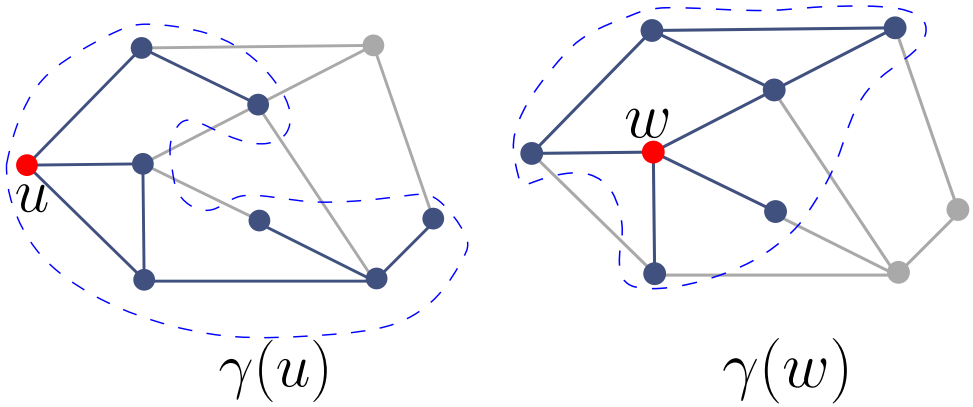


$$\mathcal{Z} = \{Z_1, Z_2, Z_3, Z_4\}$$

Initial knowledge of players

Partial knowledge model [Pagourtzis, Panagiotakos, Sakavalas, '14]

– Topology knowledge: Player u knows subgraph $\gamma(u) = (V_u, E_u)$.



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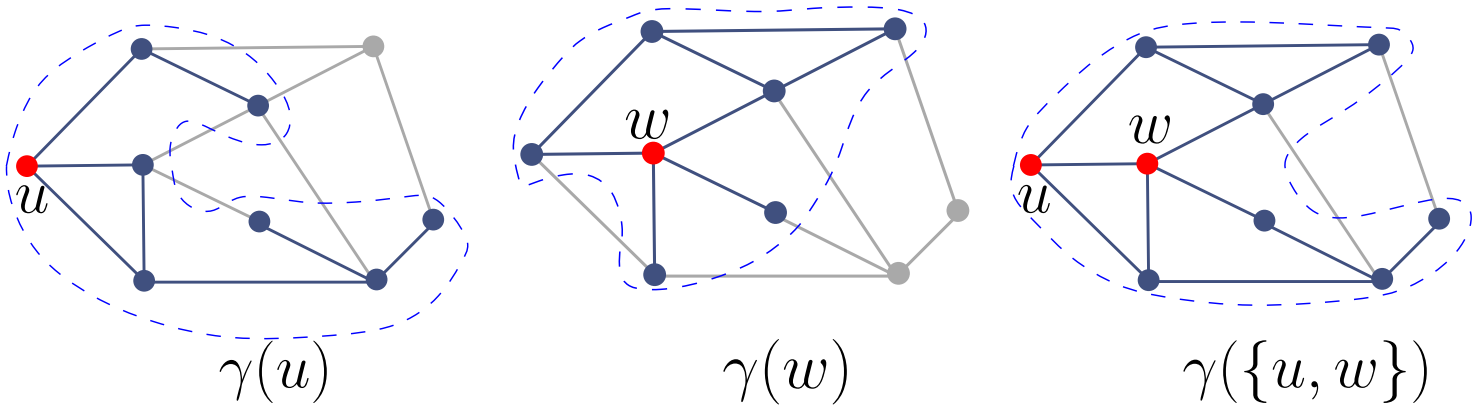

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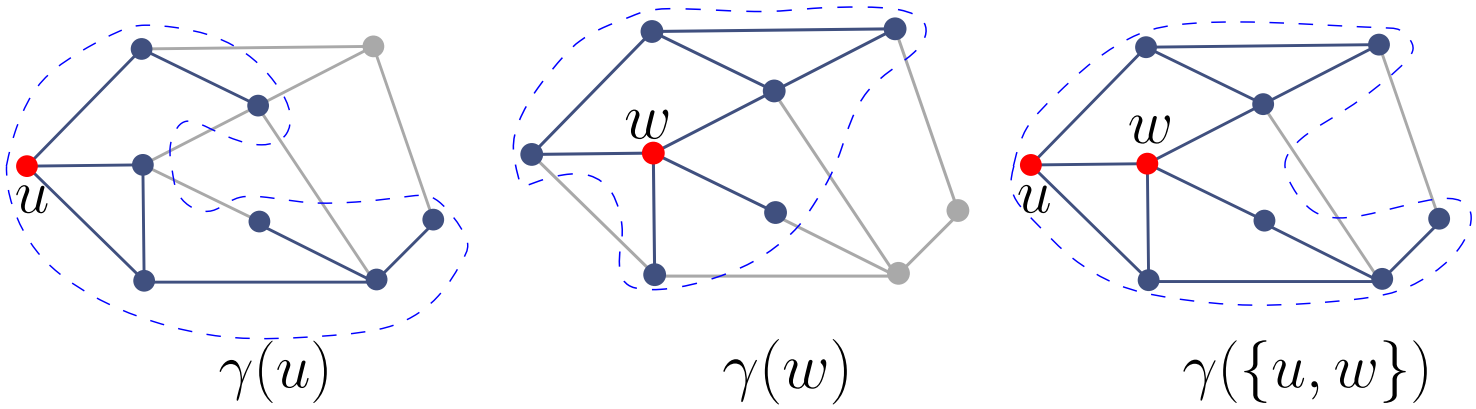

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Each player u knows only
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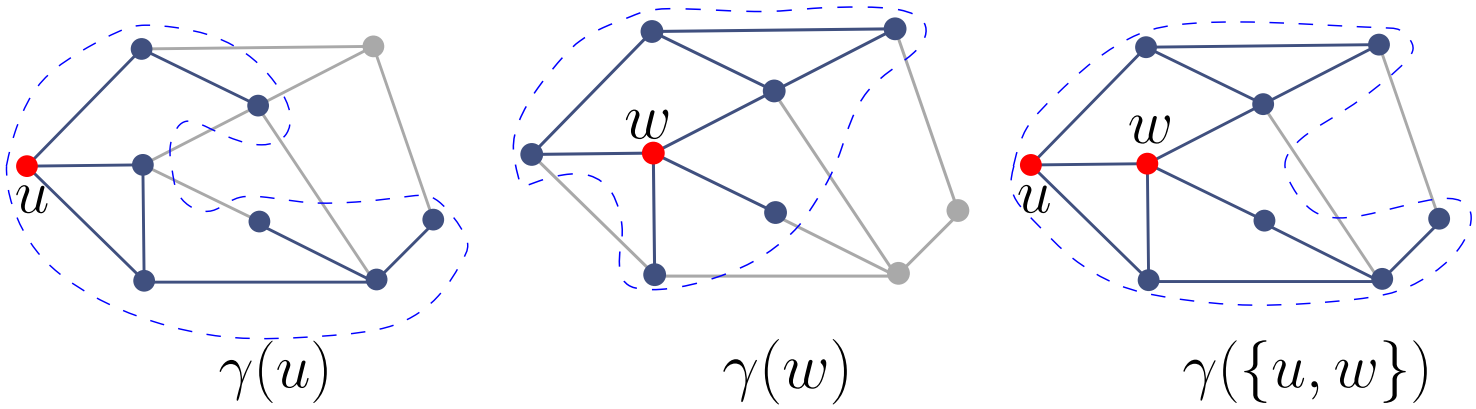

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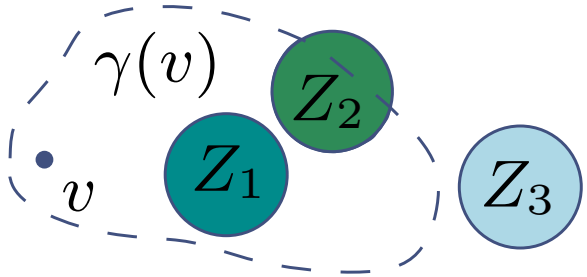

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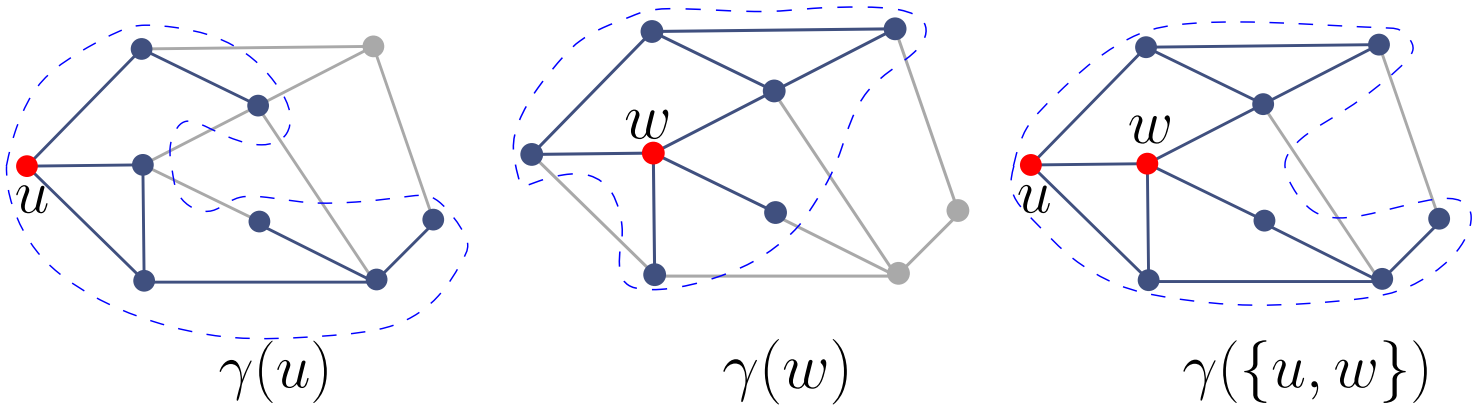

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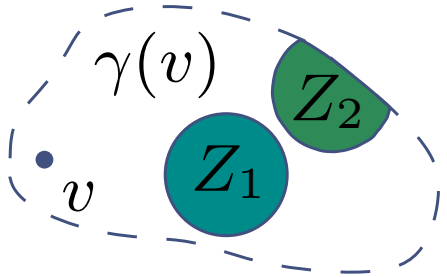

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Initial knowledge

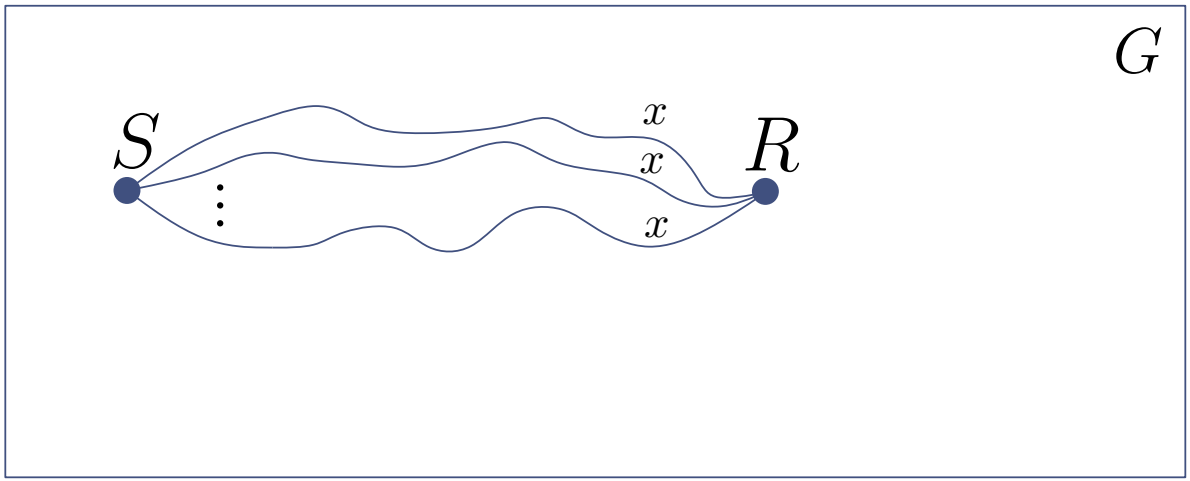
- Partial knowledge over topology and adversary.

Safe RMT algorithms [Pelc, Peleg, '05]

- Never make the receiver output (decide on) an incorrect value.

Intuition behind RMT protocols

– Known Topology: R **decides on** x upon receiving x from a set of $S \rightsquigarrow R$ paths not fully "covered" by a corruptible set.



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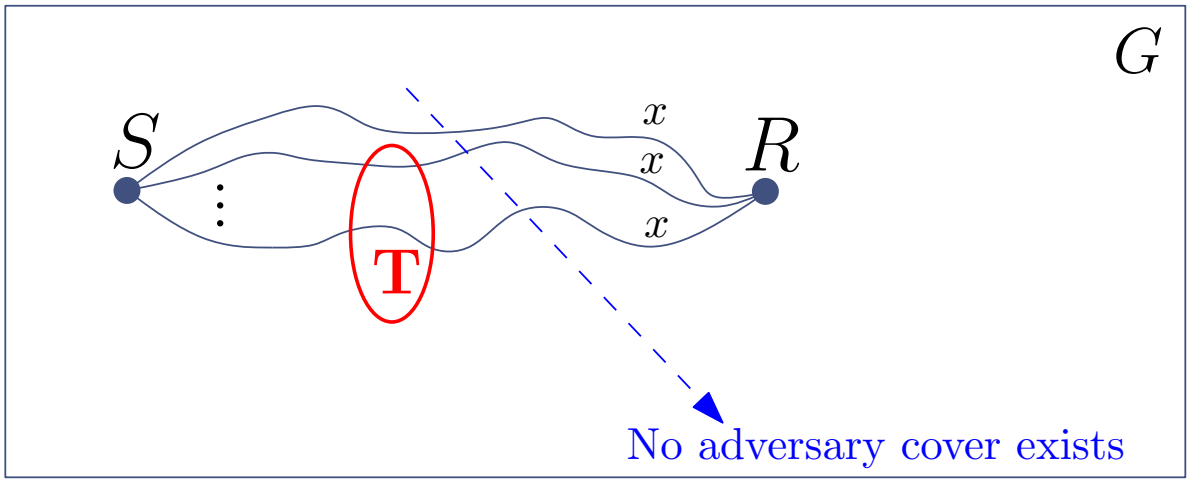

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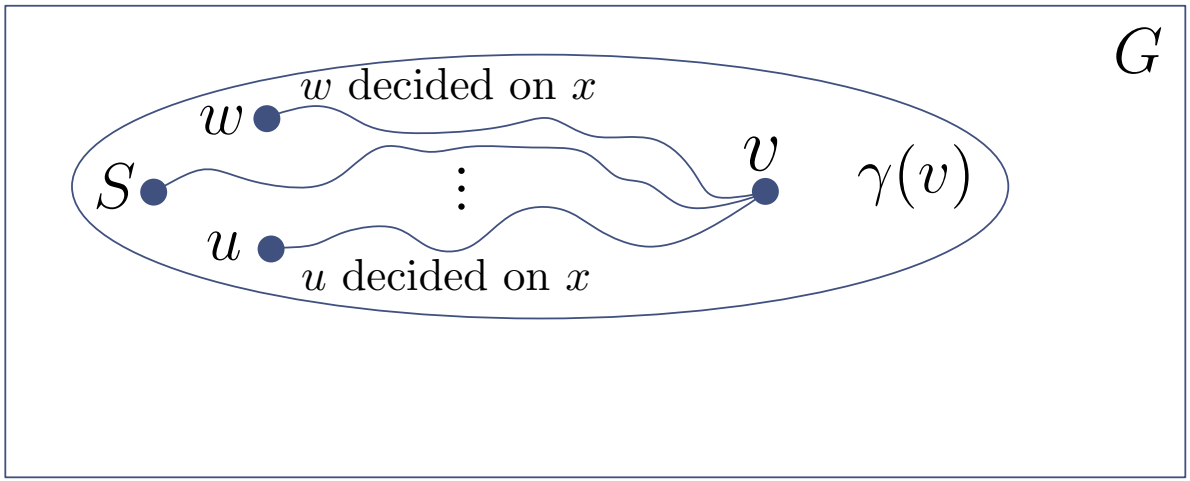

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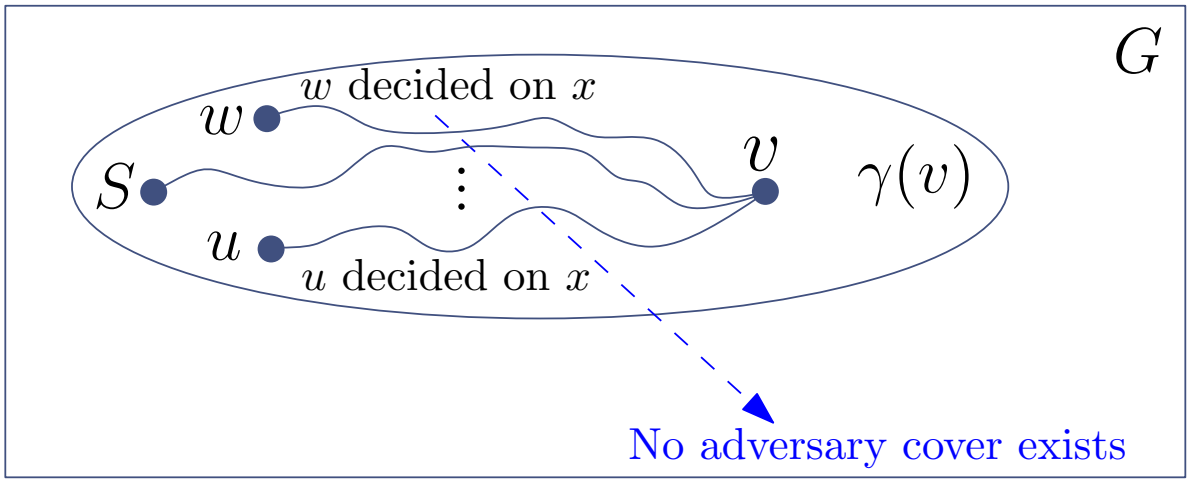

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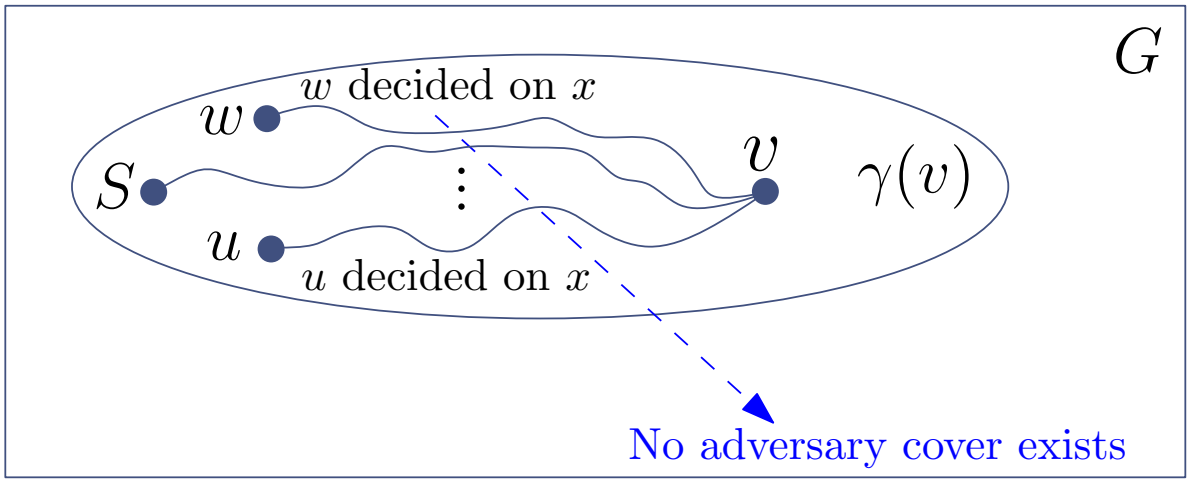

Algorithm (GPPA) tight for local knowledge [PPS14]!

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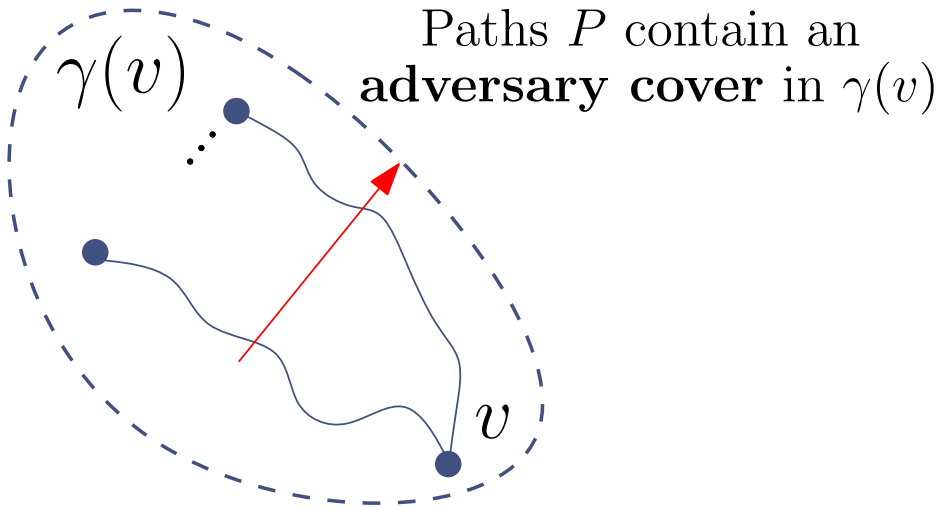

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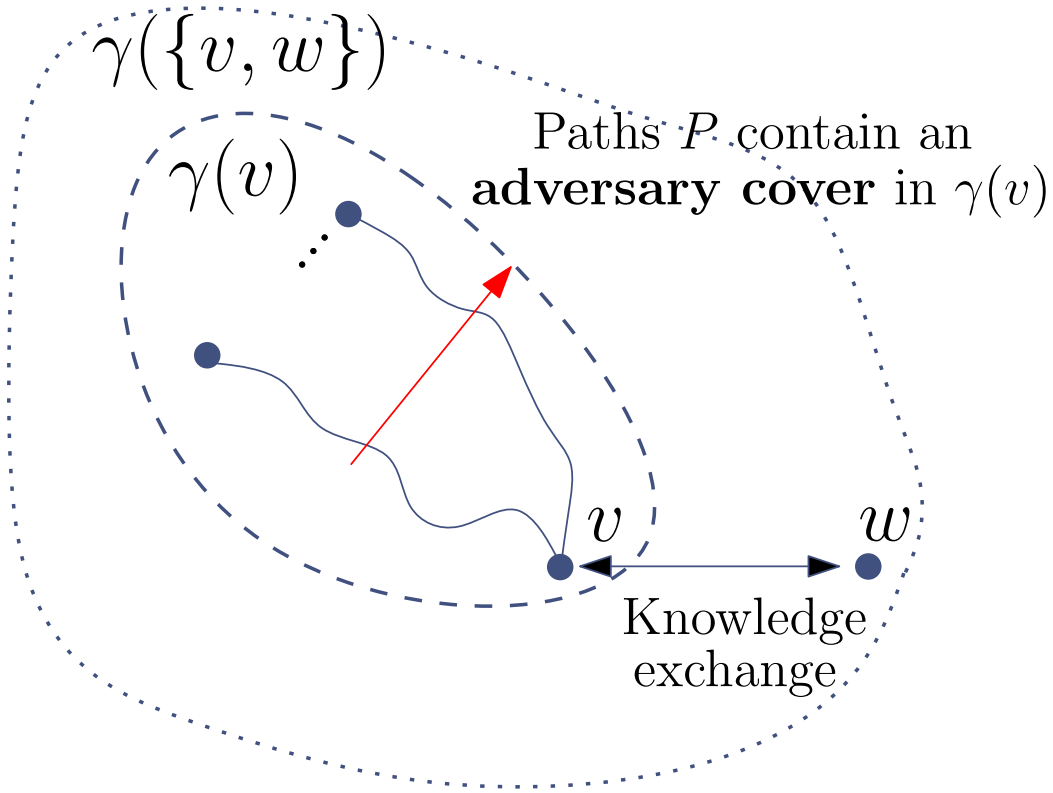

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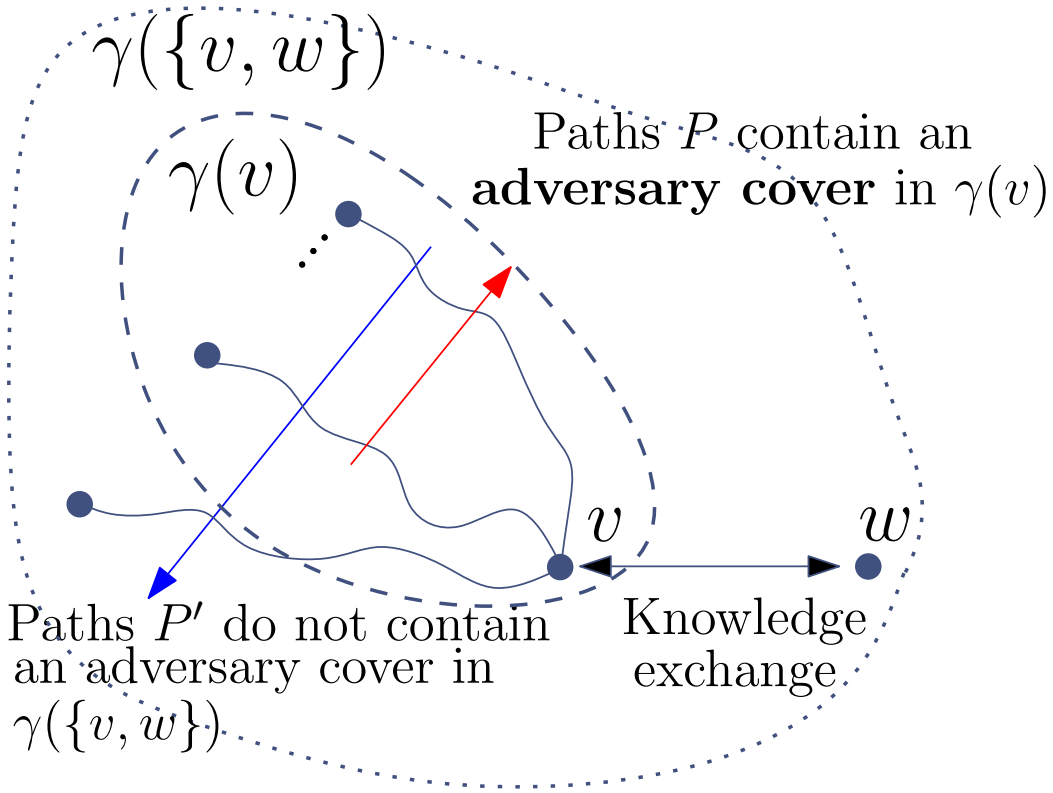

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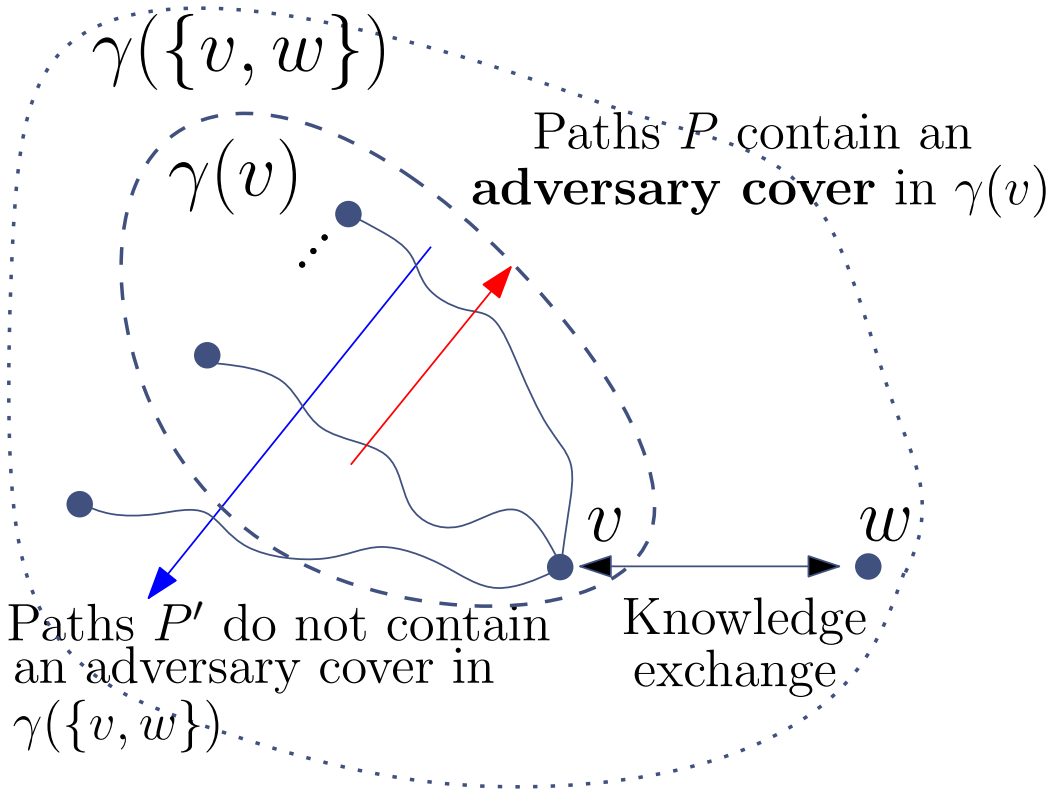

Knowledge Exchange between v, w

- **Joining topology knowledge**: trivially $\gamma(\{v, w\})$
- **Joining local adversary structures** $\mathcal{Z}_v, \mathcal{Z}_w$?

Joining Local Adversary Structures

$\mathcal{Z}^A = \{Z \cap A \mid Z \in \mathcal{Z}\}$.

For $\mathcal{Z}^A, \mathcal{Z}^B$, define (worst case) joint structure $\mathcal{Z}^A \oplus \mathcal{Z}^B$:

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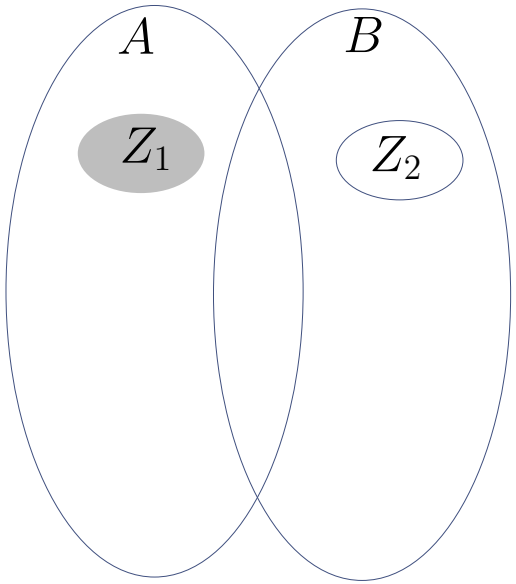

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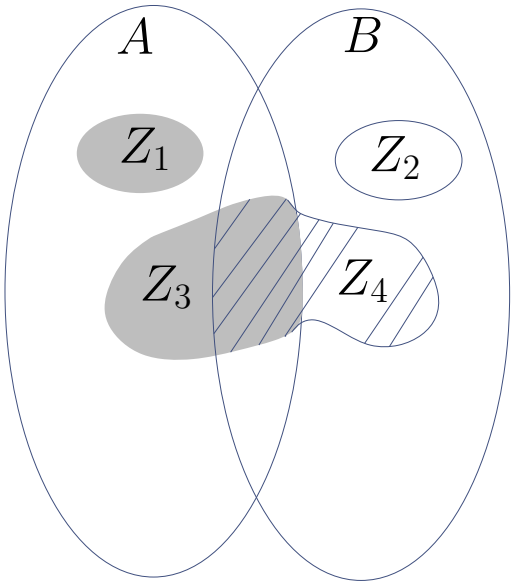

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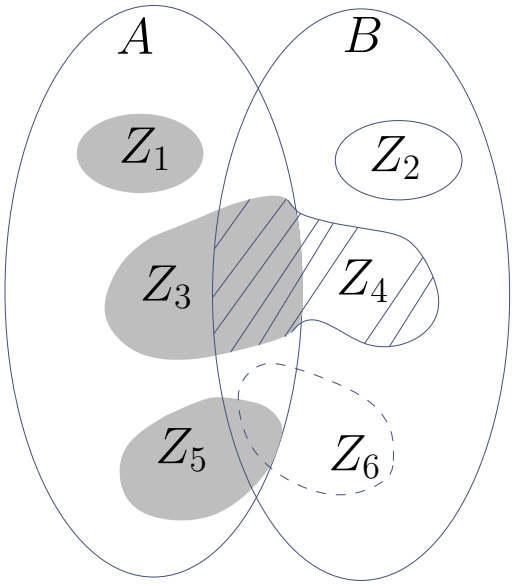

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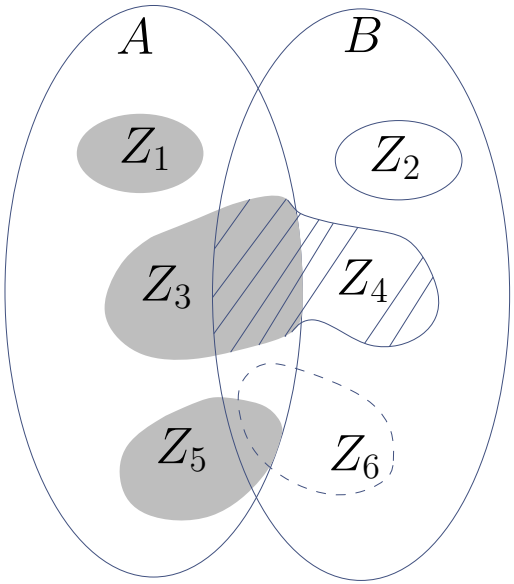

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Join Operation $\oplus$

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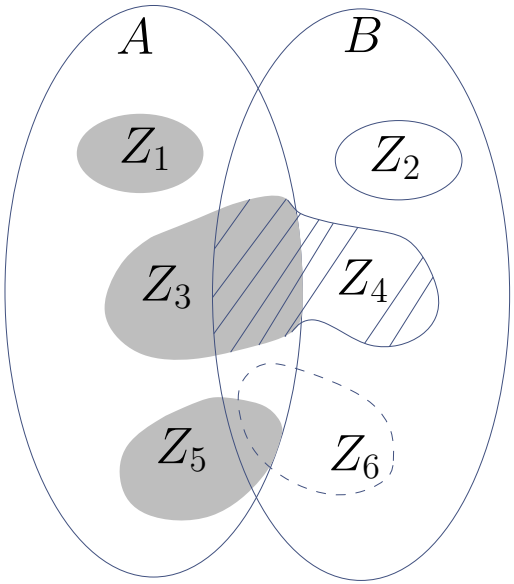

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Definition extends to different structures: $\mathcal{Z}^A \oplus \mathcal{Z}'^B$ (false structure report by adversary).

Semilattice structure of partial knowledge

Theorem

Let $\mathbb{T} = \{\mathcal{Z}^A \mid \mathcal{Z} \subseteq 2^V, A \subseteq V\}$ *the space of all possible* $\mathcal{Z}^A$.
$\langle \mathbb{T}, \oplus \rangle$ *is a semilattice.*

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Semilattices facts

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Semilattice structure of partial knowledge

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Let $\mathbb{T} = \{\mathcal{Z}^A \mid \mathcal{Z} \subseteq 2^V, A \subseteq V\}$ *the space of all possible* $\mathcal{Z}^A$. $\langle \mathbb{T}, \oplus \rangle$ *is a semilattice.*

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Semilattice structure of partial knowledgetle

Theorem (Induced partial order)

Operation $\oplus$ *induces partial order "* $\succcurlyeq$ *"* *on* $\mathbb{T}$*:*

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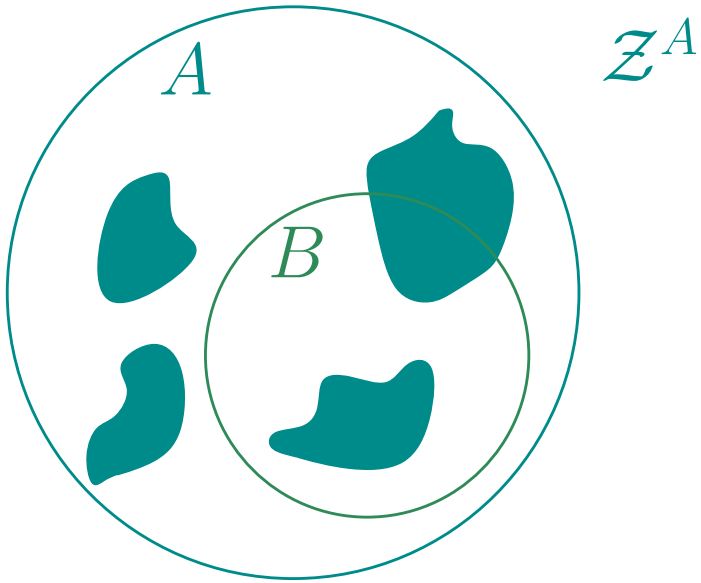

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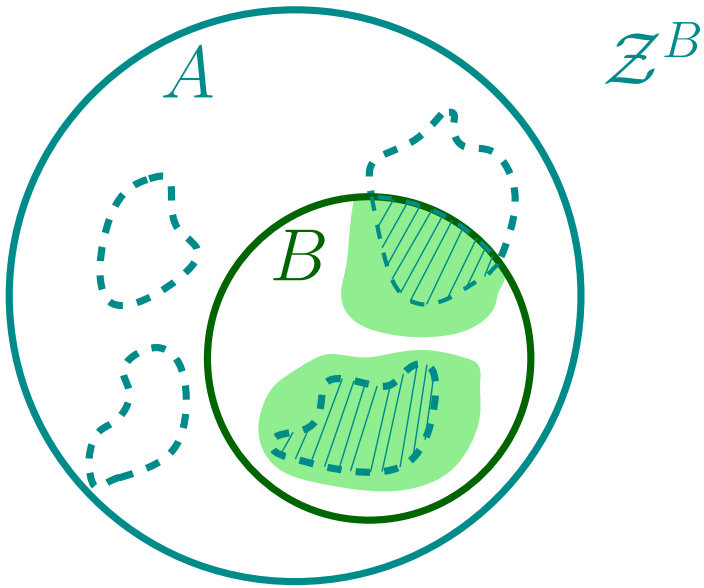

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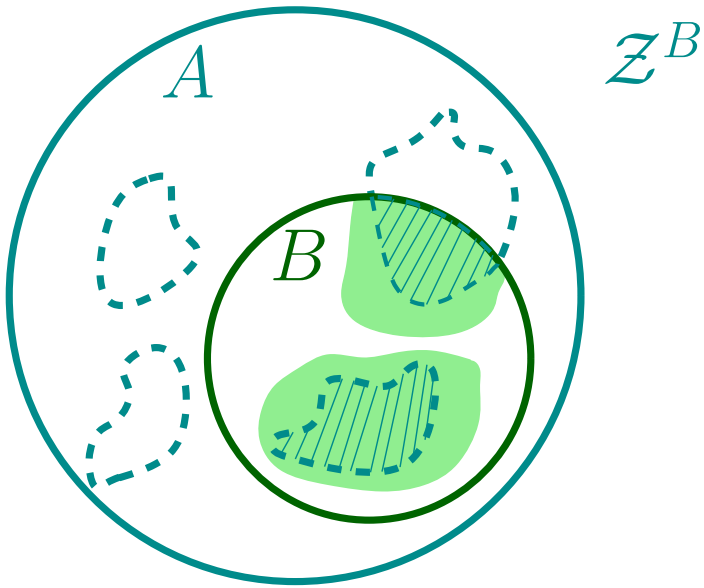
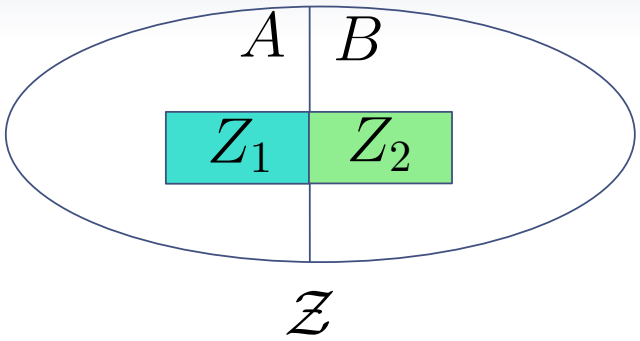
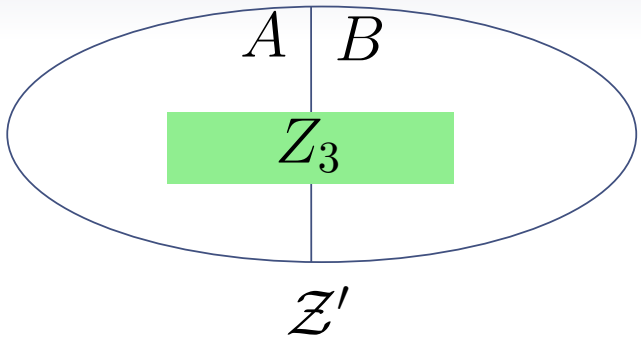


Proof. By (1) and definition of $\oplus$.

Maximality of $\mathcal{Z}^A \oplus \mathcal{Z}^B$





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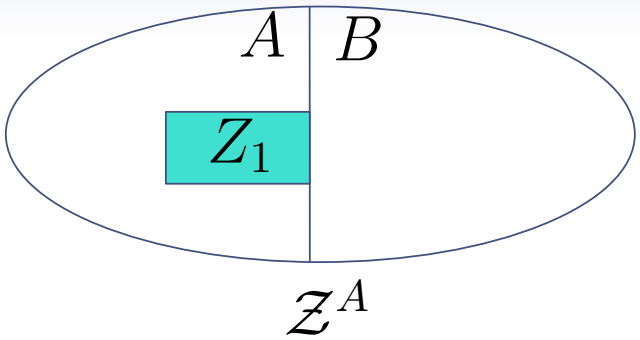

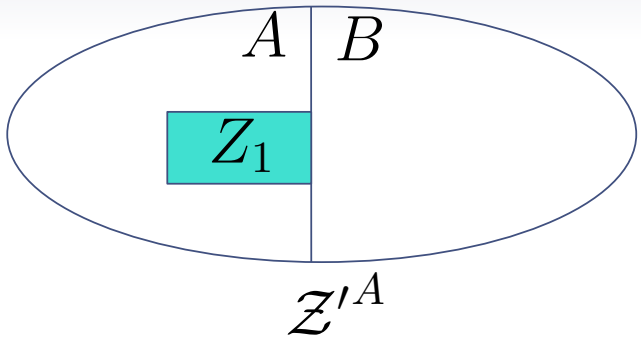

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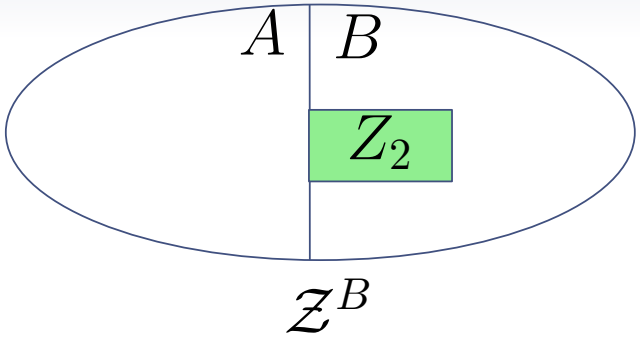

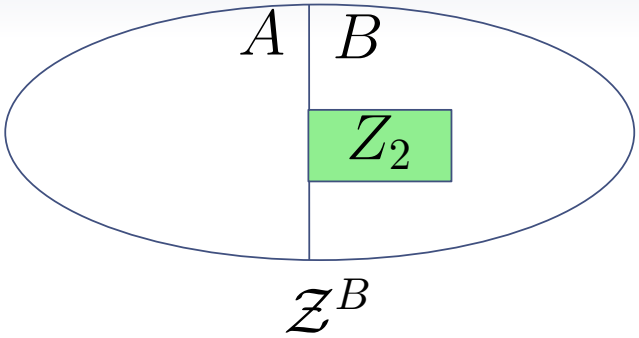

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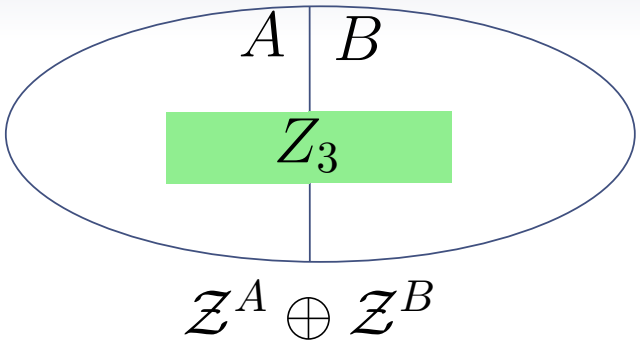

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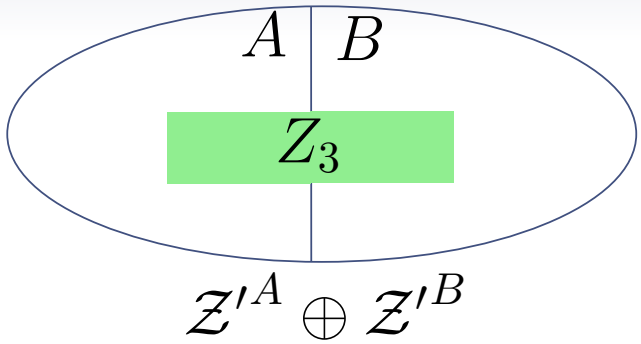

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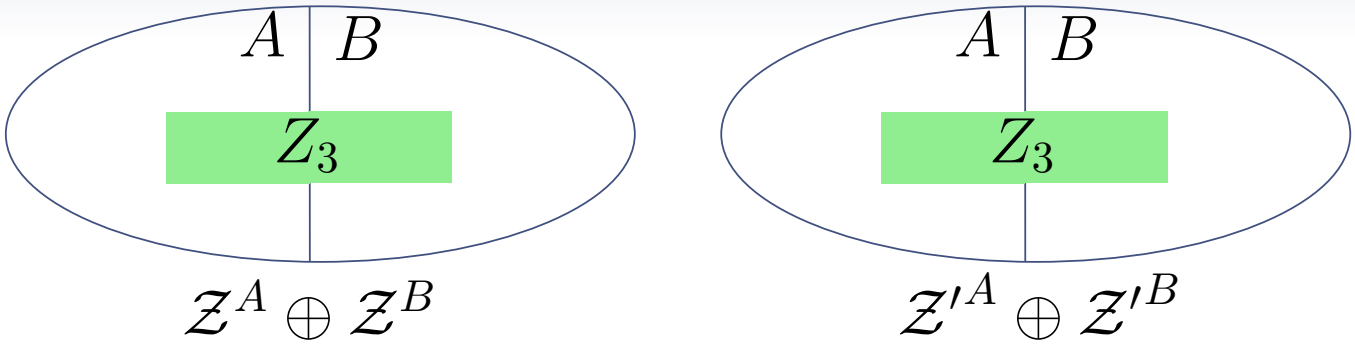

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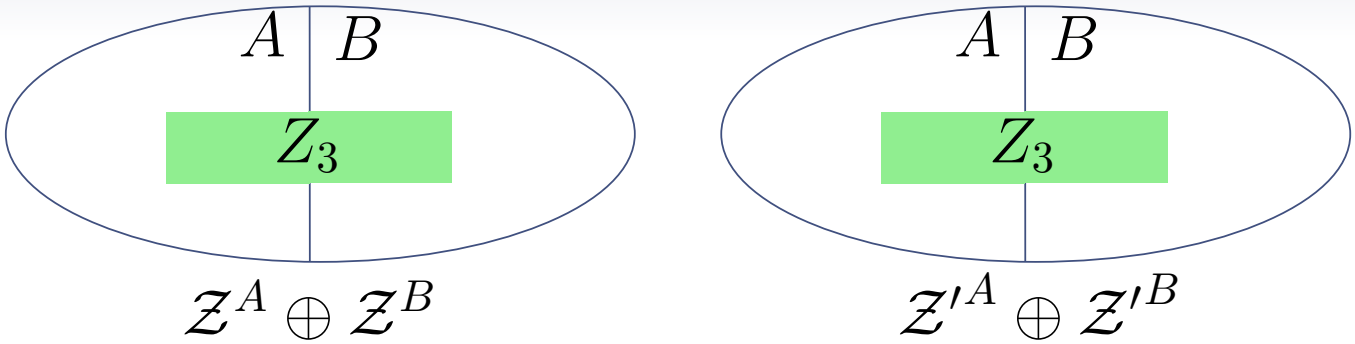

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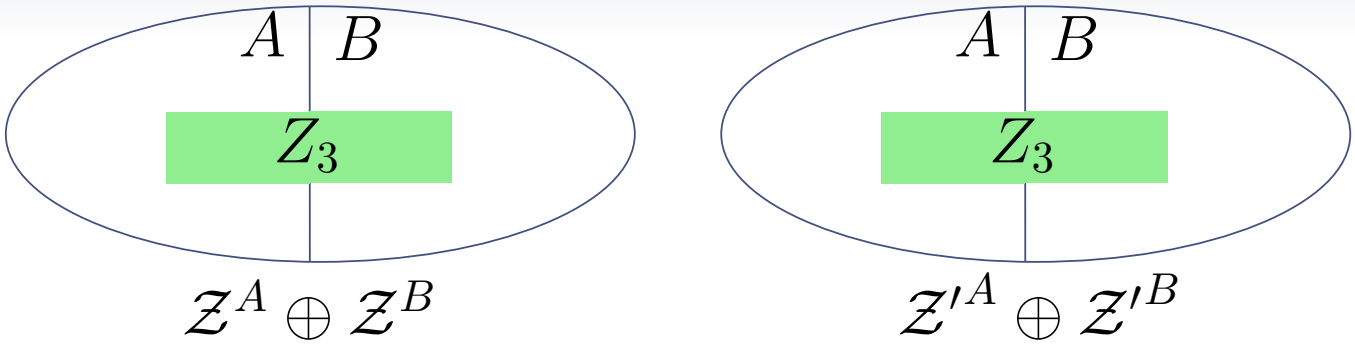

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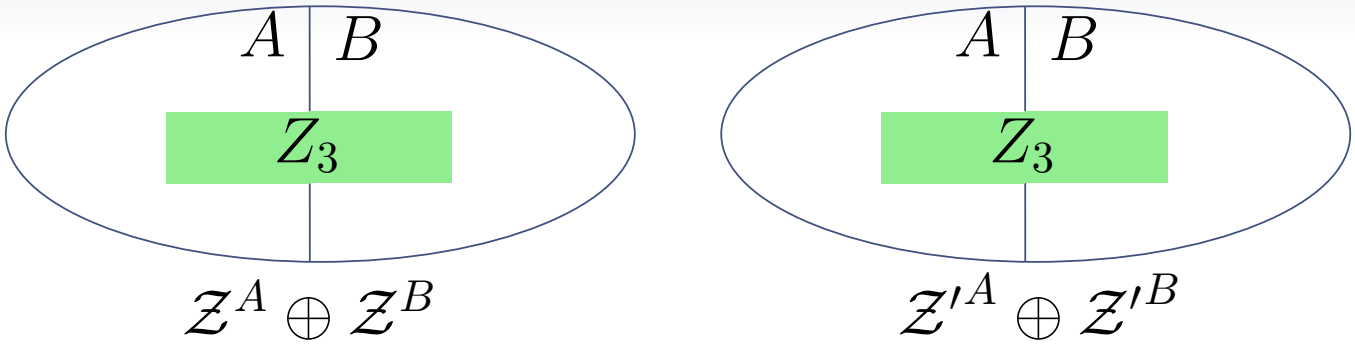

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Proof.

$$\left.\begin{array}{l} \mathcal{Z}^{(A\cup B)} \succcurlyeq \mathcal{Z}^A, \mathcal{Z}^B \\ \mathcal{Z}^A \oplus \mathcal{Z}^B = \sup\{\mathcal{Z}^A, \mathcal{Z}^B\} \end{array}\right\} \Rightarrow \mathcal{Z}^{(A\cup B)} \succcurlyeq \mathcal{Z}^A \oplus \mathcal{Z}^B \Rightarrow \ldots \quad \square$$

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Joint Adversary Structure: Maximum possible adversary structure w.r.t the initial knowledge of players in B.

$$\mathcal{Z}_B = \bigoplus_{u \in B} \mathcal{Z}^{V_u}$$

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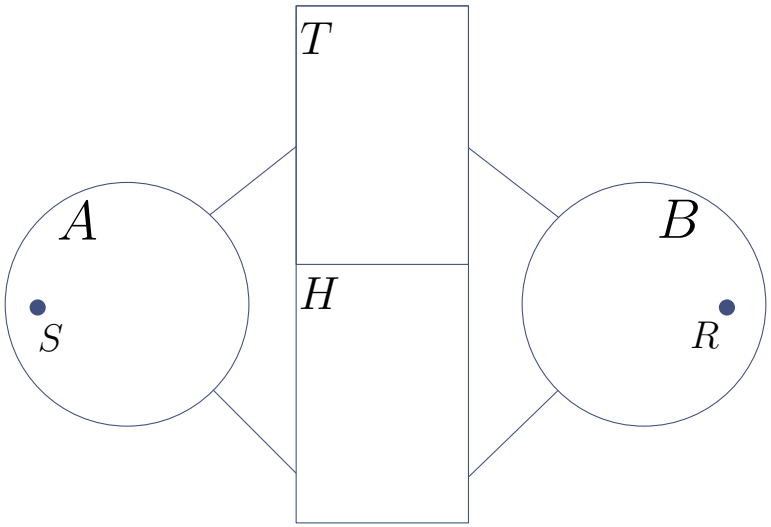

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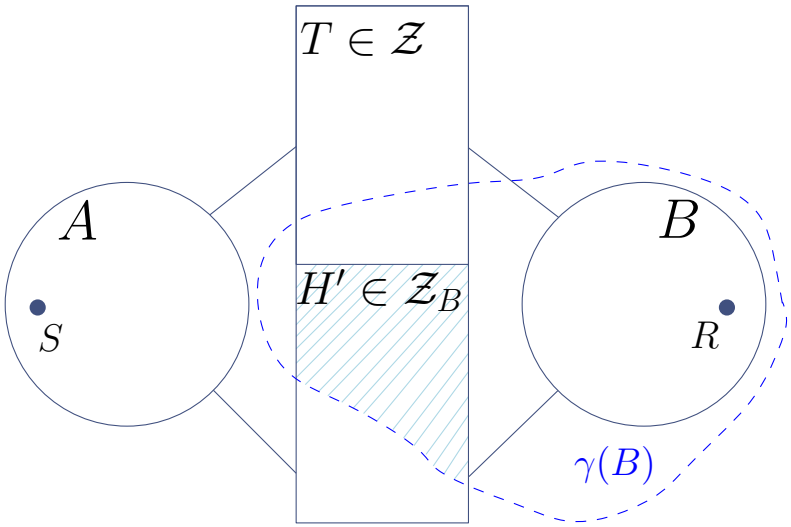

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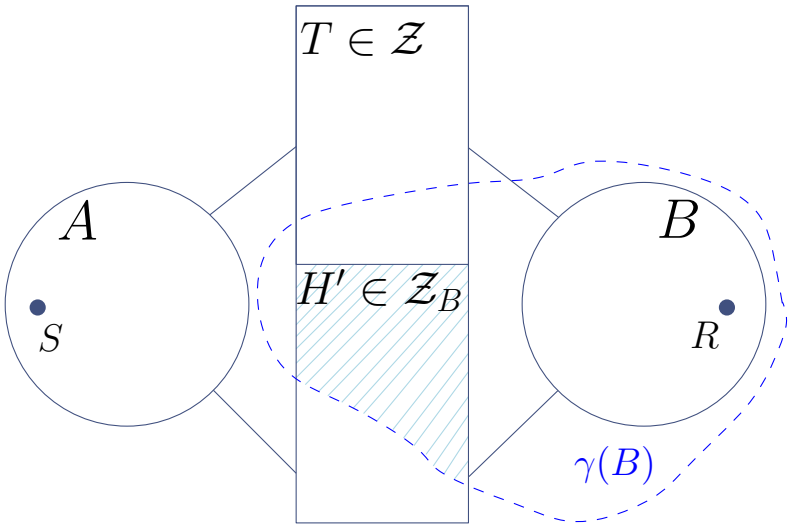

- T is corruptible.
- H "looks" corruptible to B.

RMT Impossibility

Theorem (Necessary condition for safe RMT)

If an RMT-cut exists for instance $(G, \mathcal{Z}, S, R)$ then no safe algorithm $\mathcal{A}$ can achieve RMT in $(G, \mathcal{Z}, S, R)$.

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(roughly non-safe makes assumptions that might not hold.)

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(Conflicting indistinguishable executions)

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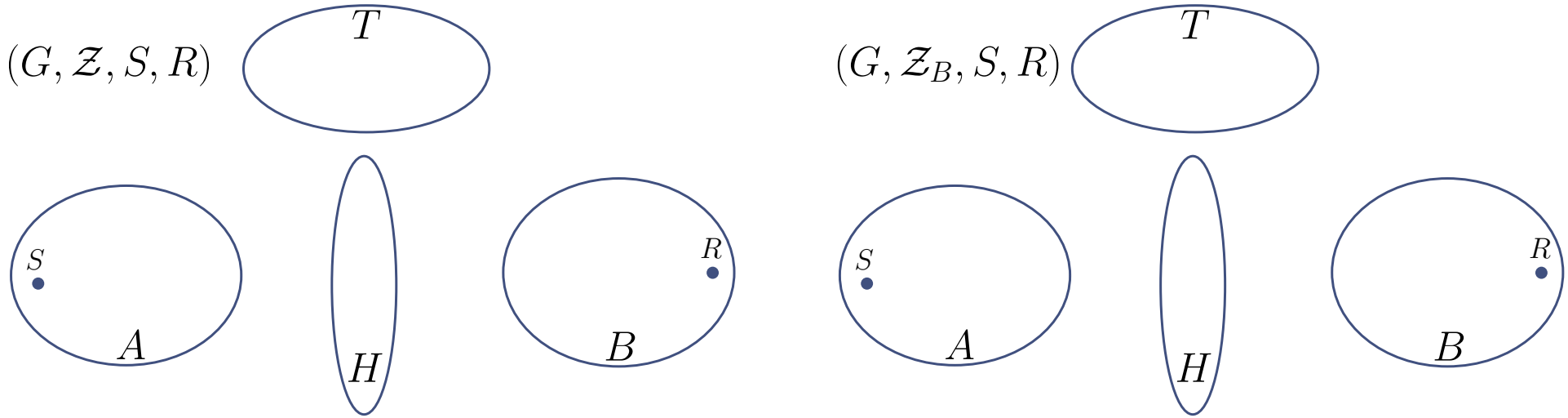

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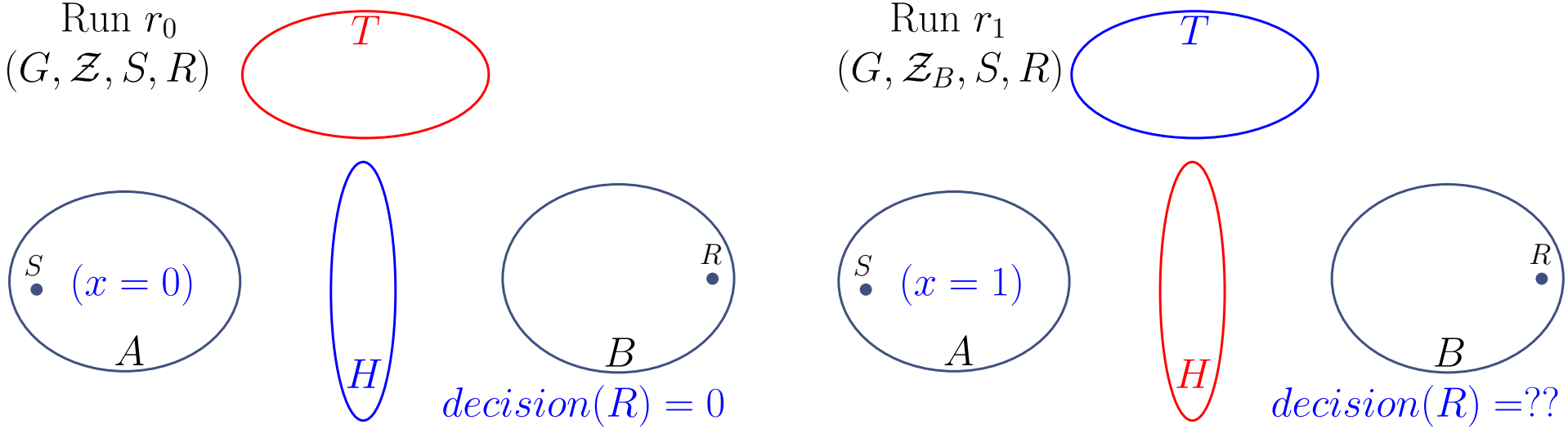

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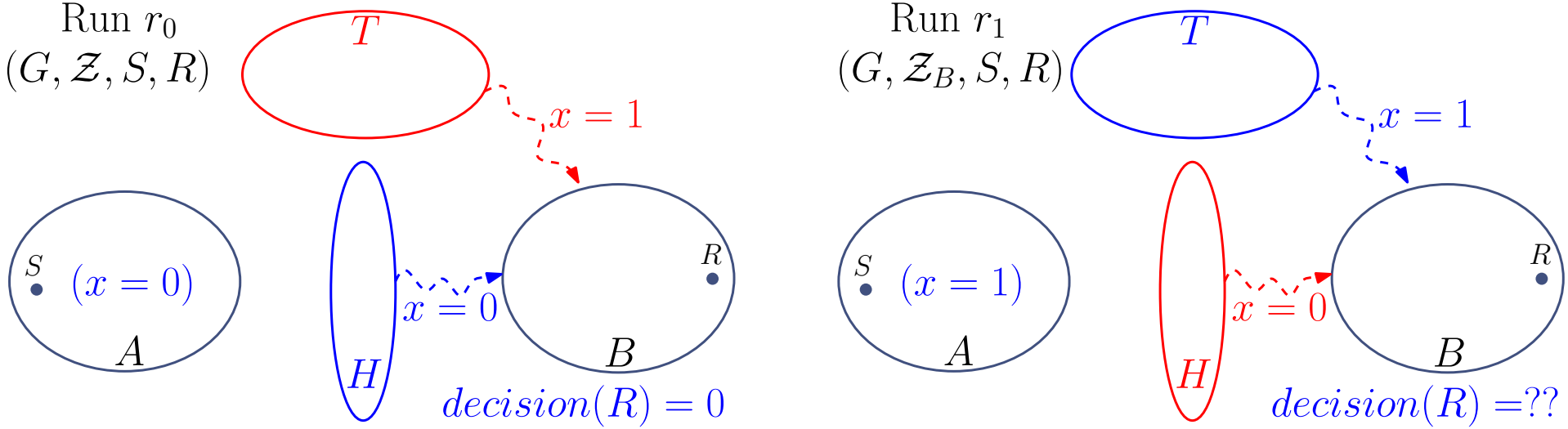

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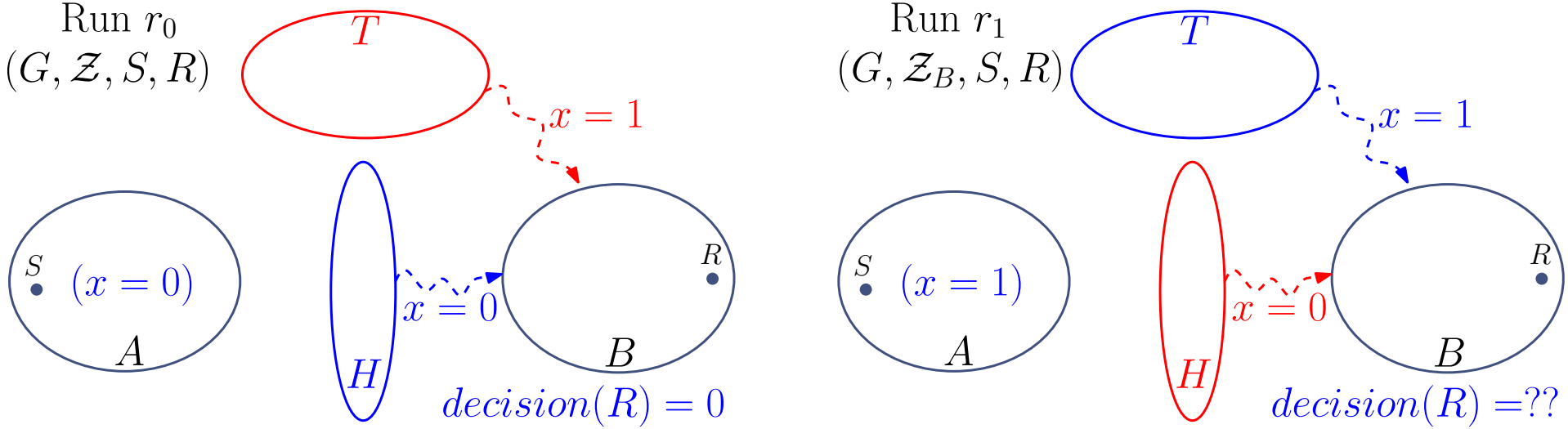

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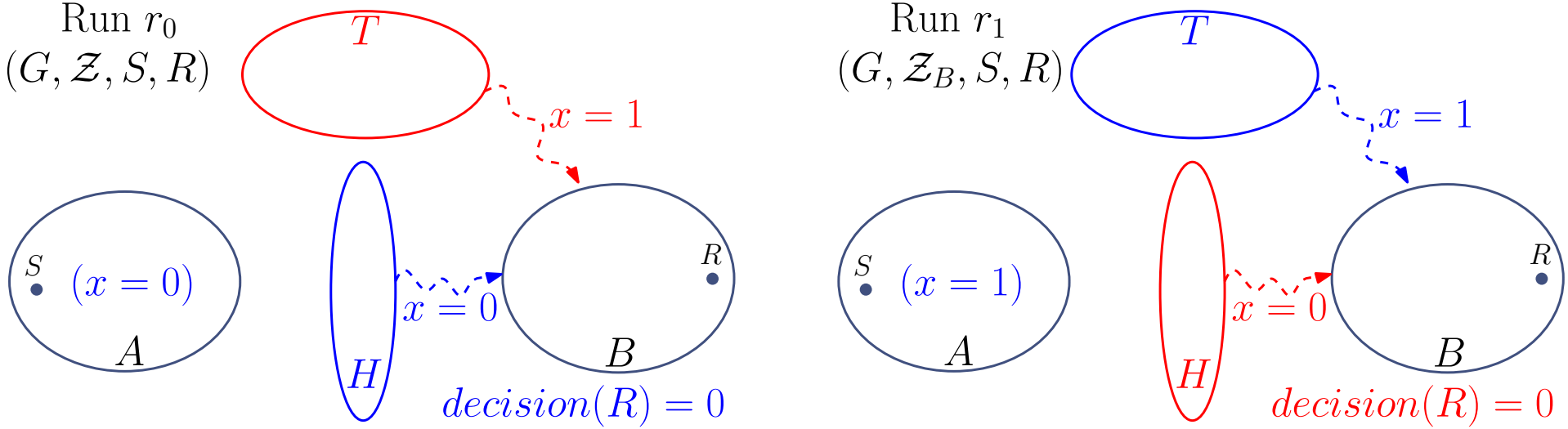

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- R decides on the same value 0 in both runs, thus $\mathcal{A}$ is not safe.

RMT-PKA Outline

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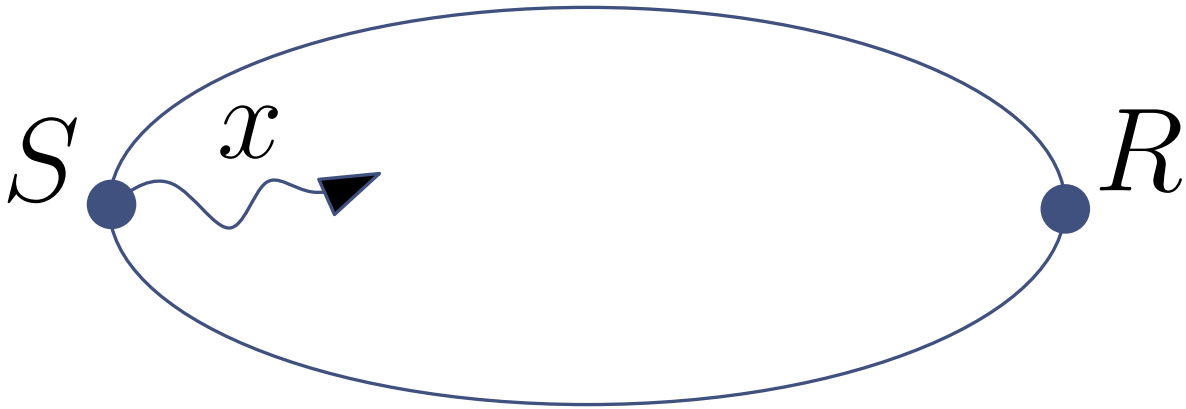
– Dealer's value is propagated throughout the graph.



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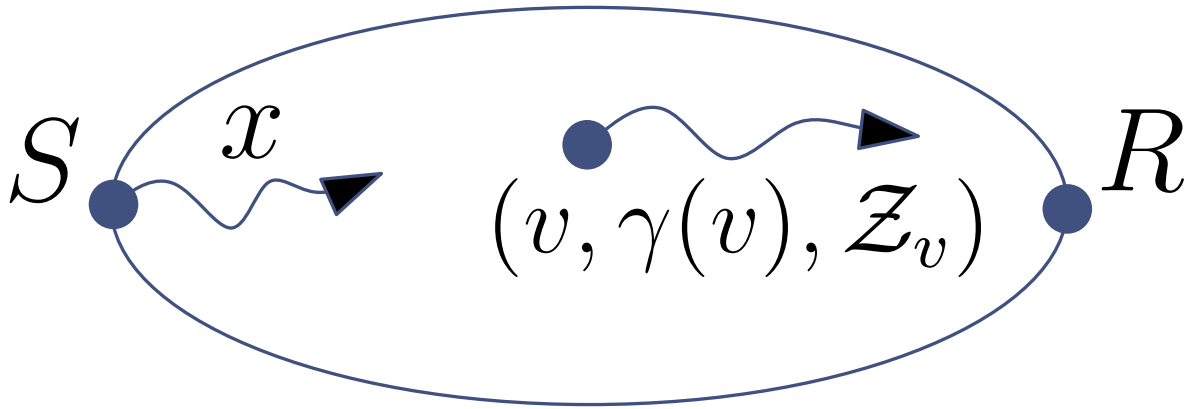

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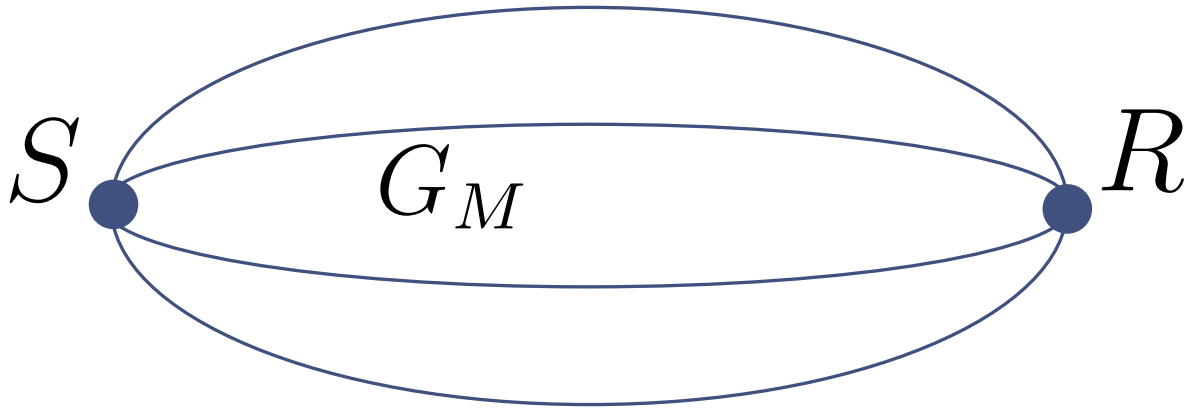

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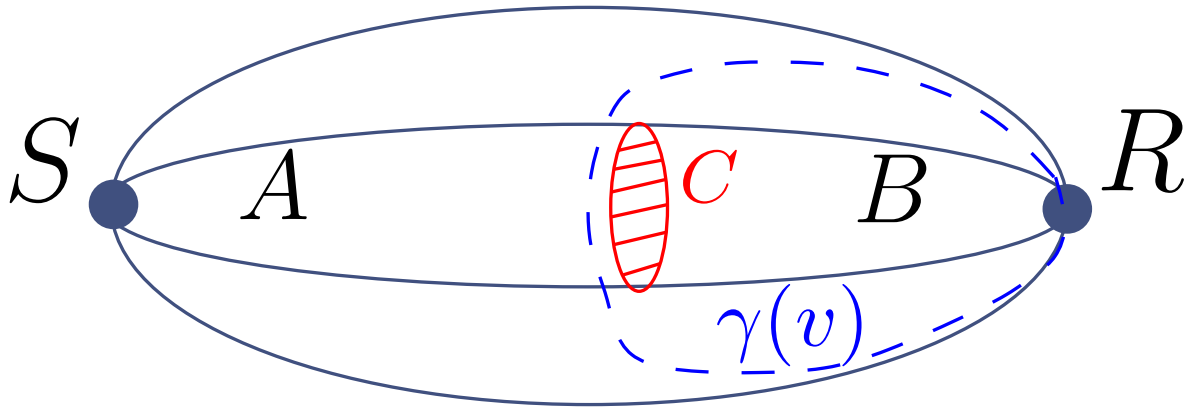

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Non-existence of an RMT-cut is a necessary and sufficient condition for achieving RMT

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- Privacy requirements in partial knowledge models (SMT).

Thank you!